BIFURCATION OF TORI FOR PARABOLIC SYSTEMS OF DIFFERENTIAL EQUATIONS WITH SMALL DIFFUSION

Klevchuk I.I.

Yuriy Fedkovych Chernivtsi National University, Chernivtsi, Ukraine

Consider a parabolic system [1–6]

$$\frac{\partial u}{\partial t} = \varepsilon D \frac{\partial^2 u}{\partial x^2} + A_0 u + \varepsilon A_1 u + F(u, \overline{u}) \tag{1}$$

with periodic condition

$$u(t, x + 2\pi) = u(t, x).$$

where ε is a small positive parameter, $u \in \mathbb{R}^n$, $D = diag(d_1, d_2, ..., d_n)$, $d_k > 0$ for $1 \le k \le n$, $A_0 = diag(i\omega_1, i\omega_2, ..., i\omega_n)$, $\omega_k > 0$ for $1 \le k \le n$, the function $F(u, \overline{u}) : \mathbb{R}^{2n} \to \mathbb{R}^{2n}$ five times continuously differentiable in u and $\overline{u}$, $F(0, 0) = 0$, $F(u, \overline{u}) = O(|u|^2$ as $|u| \to 0$.

Assume that the following condition A is satisfied:

$$p_1\omega_1 + \cdots + p_n\omega_n \ne m \quad \text{при} \quad 0 < |p_1| + \cdots + |p_n| < 6,$$

where m, p_1, $\ldots$, p_n are integer numbers.

We transform system (1) with the use of the substitution

$$u = v + \sum_{i=2}^{4} W_i(v, \overline{v}), \tag{2}$$

where W_2, W_3 and W_4 are forms of the second, third, and fourth order, respectively. Transformation (2) can be chosen so that the equation for v take the form

$$\frac{\partial v}{\partial t} = \varepsilon D \frac{\partial^2 v}{\partial x^2} + A_0 v + \varepsilon A_1 v + G(v, \overline{v})v + V(v, \overline{v}), \tag{3}$$

where $G(v, \overline{v})$ is a diagonal matrix with elements $g_k(v, \overline{v}) = \sum_{j=1}^{n} a_{kj} v_j \overline{v}_j$, $1 \le k \le n$, on the diagonal, $V(v, \overline{v}) = O(|v|^5)$ as $|v| \to 0$. For $\varepsilon = 0$ we obtain a system of ordinary differential equations. Let us assume that the zero solution of (3) for $\varepsilon = 0$ and $V(v, \overline{v}) \equiv 0$ is asymptotically stable.

Let the inequality

$$\xi_k > d_k m^2, \quad m \in \mathbb{Z}, \quad 1 \leq k \leq n,$$

be satisfied and the solution of the system

$$\varepsilon \left[\xi_k - d_k m^2\right] + \sum_{j=1}^{n} b_{kj}(\varepsilon) r_j^2 = 0, \quad 1 \leq k \leq n,$$

exist. Here $\xi_k = Re\ \mu_k$, μ_k are the diagonal elements of matrix A_1, $b_{kj} = Re\ a_{kj}$. We study existence and stability of an arbitrarily large finite number of tori.

References

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