

ON A PROBLEM OF TALAGRAND CONCERNING SEPARATELY CONTINUOUS FUNCTIONS

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ABSTRACT. We construct a separately continuous function $e : E \times K \rightarrow \{0, 1\}$ on the product of a Baire space E and a compact space K such that no restriction of e to any non-meager Borel set in $E \times K$ is continuous. The function e has no points of joint continuity and hence it provides a negative solution of Talagrand's problem in [29].

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