The Multifrequency Systems with Linearly Transformed Arguments and Multipoint and Local-Integral Conditions

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A problem of existence and uniqueness of the solution and its approximate construction for the system of differential equations with a vector of slow variables $a \in \Delta \subset \mathbb{R}^n$ and fast variables $\varphi \in T^m$ of the form

$$\frac{da}{d\tau} = X(\tau, a_\Lambda, \varphi_\Theta), \quad \frac{d\varphi}{d\tau} = \frac{\omega(\tau)}{\varepsilon} + Y(\tau, a_\Lambda, \varphi_\Theta), \tag{1}$$

are investigated in the paper. Here $\tau \in [0, L]$, small parameter $\varepsilon \in (0, \varepsilon_0]$, $0 < \lambda_1 < \dots < \lambda_p \le 1$, $0 < \theta_1 < \dots < \theta_q \le 1$, $a_\Lambda = (a_{\lambda_1}, \dots, a_{\lambda_p})$, $a_{\lambda_i}(\tau) = a(\lambda_i \tau)$, $\varphi_\Theta = (\varphi_{\theta_1}, \dots, \varphi_{\theta_q})$, $\varphi_{\theta_j}(\tau) = \varphi(\theta_j \tau)$.

Multifrequency ODE systems are researched in detail in [1], systems with delayed argument were studied in [2, 3], etc.

Conditions are set for the system (1)

$$\begin{aligned} \sum_{\nu=1}^{r} \alpha_\nu a(\tau_\nu) &= \sum_{\nu=1}^{s} \int_{\xi_\nu}^{\eta_\nu} f_\nu(\tau, a_\Lambda, \varphi_\Theta) d\tau, \\ \sum_{\nu=1}^{r} \beta_\nu \varphi(\tau_\nu) &= \sum_{\nu=1}^{s} \int_{\xi_\nu}^{\eta_\nu} g_\nu(\tau, a_\Lambda, \varphi_\Theta) d\tau, \end{aligned} \tag{2}$$

where $[\xi_\nu, \eta_\nu] \subset [0, L]$, $\bigcap_\nu [\xi_\nu, \eta_\nu] = \emptyset$.

For the problem (1), (2) a much simpler problem is constructed by averaging over fast variables φ_{θ_ν} on the cube of periods $[0, 2\pi]^{mq}$.

A sufficient condition for the system of equations (1) to exit a small circumference of the resonance of frequencies $\omega(\tau)$ was found, the condition of which in the point $\tau \in [0, L]$ is $\sum_{\nu=1}^{q} \theta_\nu \big(k_\nu, \omega(\theta_\nu \tau)\big) = 0, k_\nu \in Z^m, \|k_1\| + \dots + \|k_q\| \ne 0$.

It is proved that for the smooth enough right parts of the system (1) and subintegral functions under conditions (2), the condition to exit the circumference of resonances and for small enough $\varepsilon_0 > 0$ there exists a unique solution for the problem (1), (2) and an estimation is found

$$\|a(\tau; y, \psi, \varepsilon) - \overline{a}(\tau; \overline{y})\| + \|\varphi(\tau; y, \psi, \varepsilon) - \overline{\varphi}(\tau; \overline{y}, \overline{\psi}, \varepsilon)\| \le c_1 \varepsilon^\alpha, \alpha = (mq)^{-1},$$

where $c_1 > 0$ and does not depend on ε, $\big(a(0; y, \psi, \varepsilon), \varphi(0; y, \psi, \varepsilon)\big) = (y, \psi)$, $\big(\overline{a}(\tau; \overline{y}), \overline{\varphi}(\tau; \overline{y}, \overline{\psi}, \varepsilon)\big)$ – the solution of the averaged problem with initial conditions $(\overline{y}, \overline{\psi})$, while $\|y - \overline{y}\| + \|\psi - \overline{\psi}\| \le c_2 \varepsilon^\alpha$.

References

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