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MEAN-SQUARE STABILITY AND INSTABILITY CRITERIA FOR THE GIKHMAN–ITO STOCHASTIC DIFFUSION FUNCTIONAL DIFFERENTIAL SYSTEMS SUBJECT TO EXTERNAL DISTURBANCES OF THE TYPE OF RANDOM VARIABLES

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Abstract. *The authors investigate the asymptotic stability in quadratic mean of the trivial solution of the Gikhman–Ito stochastic diffusion functional differential equations in terms of the eigenvalues of the matrix constructed from the coefficients of these equations.*

Keywords: *criterion, stability of the solution, Gikhman–Ito stochastic functional differential equations, external disturbances.*

INTRODUCTION

Once the concept of a strong solution of a stochastic differential equation has been introduced, its properties have been analyzed, and it has been extended to the classes of stochastic functional differential equations (see the studies by Kiyosi Ito, I. I. Gikhman, A. V. Skorokhod, V. S. Korolyuk, Ye. F. Tsarkov, et al.), it became possible to analyze the asymptotic stability of the solution for stochastic functional differential equations (see, for example, [1–5]).

In the article, we will analyze the asymptotic stability in quadratic mean (in mean) of a trivial solution of the Gikhman–Ito diffusion stochastic functional differential equations (SFDE) in terms of the eigenvalues of matrix $\mathbf{B}$ composed of the coefficients of these equations.

If the maximum eigenvalue of the matrix $\mathbf{B} \max \lambda_{\mathbf{B}} < 1$, then the system is stable in quadratic mean (l.i.m.) (in mean). If $\lambda_{\mathbf{B}} > 1$, then the system is asymptotically unstable in quadratic mean. We will also analyze the case of $\lambda_{\mathbf{B}} = 1$, where $\max \lambda_{\mathbf{B}}$ is the Perron root [6] of the positive definite matrix $\mathbf{B}$.

PROBLEM STATEMENT

On a probability basis $(\Omega, F, \{F_t, t \ge 0\}, \mathbf{P})$, a diffusion SFDE is given:

$$dx(t,\omega) = \varphi(\omega)\, a(x_t)\, dt + \psi(\omega) b(x_t)\, dw(t,\omega) \tag{1}$$

under the initial conditions

$$x_t|_{t=0} \equiv \{x(t+\theta), \theta \in [-r, 0]\}|_{t=0} = \alpha \in \mathbf{R}^n, \tag{2}$$

where $x(t,\omega) \in \mathbf{R}^n$; $x_t \in \mathbf{D}_n([-r, 0])$, which is the Skorokhod space [7] of right-continuous functions with left-hand limits; $\varphi(\omega), \psi(\omega) : \Omega \to \mathbf{R}^1$ are random variables pairwise independent of n-measurable Wiener process [8]

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