SCHEMES OF APPROXIMATION OF LINEAR SYSTEMS WITH DELAY AND ANALYSIS OF THEIR STABILITY

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Differential-difference and differential-functional equations are mathematical models of many applied problems in automatic control and management systems, chemical, biological, technical, economic and other processes whose evolution depends on prehistory [1-2].

In the study of the problems of stability, oscillation, bifurcation, control, and stabilization of solutions of linear differential-difference equations, the location of the roots of the corresponding characteristic equations is very important.

This paper investigates the application of approximation schemes for differential-difference equations [3-5] to construct algorithms for the approximate finding of nonsymptotic roots of quasipolynomials and their application to study the stability of solutions of systems of linear differential equations with many delays.

Consider the initial problem for a linear system of differential-difference equations

$$\frac{dx}{dt} = Ax(t) + \sum_{i=1}^{k} B_i x(t-\tau_i), \qquad (1)$$

$$x(t) = \varphi(t),\ t \in [-\tau, 0], \qquad (2)$$

where A, B_i, $i=\overline{1,k}$ fixed $n \times n$ matrix, $x \in R^n$, $0 < \tau_1 < \tau_2 < ... < \tau_k = \tau$, $\varphi(t) \in [-\tau, 0]$.

Let us correspond to the initial problem (1) - (2) the system of ordinary differential equations [4-5]

$$\begin{aligned} \frac{dz_0(t)}{dt} &= A(t)z_0(t) + \sum_{i=1}^{k} B_i z_{l_i}(t),\ l_i = [\frac{\tau_i m}{\tau}], \\ \frac{dz_j(t)}{dt} &= \mu(z_{j-1}(t) - z_j(t)),\ j = \overline{1,m},\ \mu = \frac{m}{\tau},\ m \in N, \end{aligned} \qquad (3)$$

with initial conditions

$$z_j(0) = \varphi(-\frac{\tau j}{m}),\ j = \overline{0,m}. \qquad (4)$$

Theorem [4]. *If the zero solution of the system with delay (1) is exponentially stable (not stable), then there is $m_0>0$ such that for all $m>m_0$, the zero solution of the approximating system (3) is also exponentially stable (not stable).*

If for all $m>m_0$ the zero solution of the approximation system (3) is exponentially stable (not stable) then the zero solution of the system with a delay (1) is exponentially stable (not stable).

It follows from Theorem 2 that the asymptotic stability or instability of the solutions of the delayed linear equations and the corresponding approximating system of ordinary differential equations for sufficiently large values of m are equivalent.

This allowed us to build methods for studying the stability of linear systems with many delays, which can be constructively implemented on a computer using Mathematica software or builtin Python libraries. Computational experiments on special test examples showed the high efficiency of the proposed algorithms for studying the stability of linear differential-difference equations.

References

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