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Plenary talks

Evoluţii şi tendinţe în studierea ştiinţelor reale în Republica Moldova

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În ultimele decenii se prefigurează o nouă concepție privind existența umană. În Recomandarea Consiliului Europei din 22 mai 2018 privind competențele-cheie pentru învățarea pe tot parcursul vieții (*Jurnalul Oficial al Uniunii Europene, 2018/C 189/01*) se formulează abilitățile necesare pentru un mod de viață în continuă schimbare, care necesită adaptare și învățare permanentă. Evidențiem în acest sens: *promovarea dobândirii de competențe în științe (S), tehnologie (T), inginerie (E) și matematică (M)* (STEM)*, ținând seama de legăturile acestora cu artele, creativitatea și inovarea și motivând mai mulți tineri, în special fete și tinere femei, să îmbrățișeze o carieră în domeniile STEM.*
Este cunoscut însă faptul că tinerii manifestă un interes scăzut pentru studierea disciplinelor din domeniul științelor reale și ale naturii, fapt ce duce la o lipsă acută de cadre calificate atât în domeniul educației, cât și în sectoarele economiei reale.
În scopul redresării situației, în mai multe țări dezvoltate, în sistemul educațional se implementează conceptul STEAM, care presupune studierea integrată a mai multor discipline reale, în special, a științei (S), a tehnicii (T), a ingineriei (E) și a matematicii (M). În scopul dezvoltării armonioase a elevilor, în acest concept a fost inclusă și arta (A). În prezent se promovează tot mai intens implementarea conceptului STREAM, care este integrarea STEAM cu adăugarea lui R pentru citire și scriere, ajutând elevii/studenții să comunice mai eficient, ceea ce este un aspect important al interacțiunilor umane.
Astfel, interdisciplinaritatea nu trebuie concepută numai în sensul integrării cunoştinţelor, ci şi ca mod de gândire şi acţiune. Perspectiva interdisciplinară constă în esenţă în familiarizarea elevilor/studenților cu principii, cunoștințe și metode generale interdisciplinare, care ar putea fi aplicate în contexte cât mai diverse posibil pentru soluționarea problemelor reale.
Mai jos ne vom referi la unele rezultate obținute de echipa de cercetători din cadrul Univesității de Stat din Tiraspol realizate în cadrul studiului monografic ”Evaluarea procesului de studiere a științelor reale și ale naturii din perspectiva inter/transdisciplinarității (concept STEAM)”
Tendințe îngrijorătoare pentru Republica Moldova
S-a constatat că majoritatea elevilor nu sunt incluse în iniţiativele de încurajare a motivaţiei către alegerea carierelor STEAM, previziunile arătând că, în viitorul apropiat, vom avea tot mai puțini specialiști și tinerii nu vor alege cariere vitale pentru dezvoltarea economiei.
Conform statisticii oficiale, în Republica Moldova, tot mai puțini elevi optează pentru profilul real. Numărul absolvenților claselor de liceu cu profil real, în ultima perioadă, a scăzut simțitor. Și astfel, studenții înmatriculați la ciclul I și ciclul II, în cea mai mare parte, nu doresc să-și continue studiile în domeniile care au conexiune cu studierea profundă a științelor reale.
Care sunt cauzele? Una dintre cauzele principale, în acest sens, ține de sistemul educațional care îi izolează pe elevi/studenți de problemele reale. Elevii/studenții nu conștientizează de ce le-ar trebui învățarea științelor reale, nu văd aplicarea lor în viață, nu înțeleg utilitatea disciplinelor reale, nu sesizează conexiunea lor cu fenomenele și procesele economice. Pentru a schimba lucrurile, obiectivele învățării științelor reale trebuie să fie axate pe flexibilitate, variație și implementarea TIC-ului (tehnologiile informaționale și comunicaționale) în procesul de predare-învățare pentru a-i pregăti pe tineri să folosească cunoștințele, metodele științifice și tehnologiile informaționale în mod creativ, în viața reală, dintr-o perspectivă inter/transdisciplinară.

Situația actuală privind procesul de studiere a științelor reale
În cercetările realizate au fost scoase în evidență schimbările structurale în derularea acestui proces. În anii 2010-2017, numărul de candidați pentru susținerea examenului de BAC, la profilul real, a fost în permanență mai mare comparativ cu numărul de candidați de la profilul umanist. **În anul 2018, în premieră, s-au produs schimbări structurale sub acest aspect**. Astfel, în anii 2018 și 2019 numărul de candidați de la profilul umanist a depășit cu circa 120 și, respectiv, cu 693 de persoane numărul de candidați din anii precedenți. **Conform previziunii autorilor, această tendință se va menține pe parcursul următorilor ani**.
Pe parcursul anilor 2010-2019, cei mai mulți candidați înregistrați pentru a susține examenul de bacalaureat (profilurile real, umanist, arte, sport și tehnologie) s-au înregistrat în anul 2011, circa 29995 de persoane. Iar cei mai puțini candidați, într-un număr de 17165 de persoane, au fost în anul 2019. Conform datelor existente, numărul total de candidați în anul 2019 a scăzut cu circa 12830 de persoane, sau de circa 1,8 de ori comparativ cu anul 2011. Astfel, de exemplu, la profilul real numărul de pretendenți s-a micșorat în anul 2019 comparativ cu anul 2011 cu circa 8062 de persoane, iar numărul de elevi la profilul umanist - cu aproximativ 4238 de persoane.
Iar rata de promovare, la susținerea examenelor de BAC, pe parcursul anilor 2016-2019, pentru candidații de la profilul real este cu 30% mai mică decât a celor de la profilul umanist.
Performanțele academice ale olimpicilor moldoveni
Au fost examinate rezultatele atât la concursuri naționale și internaționale, cât și activitatea profesională ale olimpicilor moldoveni la informatică, matematică, fizică, chimie, biologie și geografie pe perioada 2015-2019. În cea mai mare parte olimpicii moldoveni au o pregătire foarte bună la disciplinele pe care le reprezintă și acest fapt le permite să înregistreze rezultate frumoase nu numai la olimpiadele naționale, dar și la cele internaționale, cu excepția Olimpiadei Internaționale la geografie, deoarece echipa Republicii Moldova nu participă.
Problema cea mai mare, în opinia noastră, este că majoritatea olimpicilor care s-au remarcat pleacă la studii peste hotare și ulterior, după absolvire, foarte puțini intenționează să revină în țară, producându-se astfel un exod masiv de talente.
În acest sens, de exemplu, olimpicii la informatică în proporție de circa 75%, în anii 2015-2019, au ales să-și continue studiile peste hotare. Astfel: 45% optează pentru universitățile din România, 8% aleg Federația Rusă, 6% merg la studii în SUA, iar restul 16% își continuă studiile la alte universități de pe mapamond. În Republica Moldova rămân circa 25% dintre absolvenții olimpici.
Doar 9-10% dintre olimpicii la matematică decid să rămână în Republica Moldova. Cam 21,2% dintre olimpicii la matematică aleg să plece în România și cam același procent, aproximativ 21,2%, merg la universitățile din SUA. În universitățile europene (Germania, Franța, Elveția, Italia etc.) decid să meargă circa 25% dintre olimpici.
Dintre olimpicii la fizică au ales să studieze în continuare în universitățile din UE - 74% și din SUA - 22%. În Moldova, pentru continuarea studiilor a optat, în perioada 2015-2019, doar un singur olimpic.
Olimpicii la chimie în proporție de 75,5%, au optat preponderent pentru continuarea studiilor în instituții superioare de învățământ de peste hotarele Republicii Moldova. Dintre care: 40,8% - România, 14,3% - Franța, câte 4,08% - Federația Rusă, SUA, Olanda și Marea Britanie, restul țărilor câte 2,04%.
Cei mai mulți olimpici la biologie au decis să-și continue studiile la universitățile din România - 43%, în Moldova au rămas circa 27% dintre olimpici, în SUA - 12%, iar restul 18% preferă continuarea studiilor la universități din Europa.
Din olimpicii la geografie, aproximativ 32% au hotărât să studieze la universitățile din țară. Restul sunt pentru universitățile din România (55%) și Europa (13%).
Implementarea conceptului de inter/transdisciplinaritate în învățământul preuniversitar
Au fost examinate diverse abordări inter/transdiciplinare în procesul de studiere a științelor reale și

ale naturii. Modelele inter/transdiciplinare descrise pot fi utilizate pe larg la predarea informaticii, matematicii, fizicii, chimiei, biologiei și geografiei.
În sondajul realizat de echipa de implementare a proiectului, desfășurat pe un eșantion de 234 de profesori din domeniul științelor reale și ale naturii, au fost incluse și întrebări ce solicită părerea respondenților vizavi de implementarea conceptelor de inter- și transdisciplinaritate în procesul de studiere a disciplinelor predate. Astfel, la întrebarea dacă practică instruirea interdisciplinară/transdisciplinară în procesul didactic, au fost obținute următoarele rezultate: 68,4% susțin că implementează conceptul de interdisciplinaritate, iar aproximativ 29% afirmă că implementează parțial și 1,7% nu practică o astfel de instruire. La întrebarea dacă implementează conceptul STEM/STEAM în cadrul orelor (instruirii), doar 29,1% dintre intervievați au afirmat acest lucru.
S-a constat faptul că circa 15,7% dintre respondenți nu implementează conceptul STEAM, 47% îl implementează doar parțial, iar 8,3% sunt doar la nivel de intenție.

Studierea științelor reale și continuarea studiilor în învățământul superior

Au fost examinate tendințele care se conturează în învățământul superior din perspectiva studierii celor 10 domenii fundamentale la ciclul I și ciclul II.
În acest sens s-a punctat faptul că din cele 10 domenii fundamentale, în anul 2018 și 2019, cele mai puțin solicitate de către absolvenții de liceu sunt „Științe ale naturii, matematică și statistică" (2,1% și respectiv 2%) și „Științe agricole, silvicultură, piscicultură și medicină veterinară" (1,6% și respectiv 1,8%). Salariile foarte mici ale specialităților care țin de aceste domenii de mare importanță pentru economia națională nu reprezintă o atracție pentru tinerii din ziua de azi. Numai așa poate fi explicat procentul destul de scăzut al absolvenților de liceu care doresc să studieze disciplinele legate de aceste domenii atât de necesare pentru dezvoltarea țării.

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Parameter identification in models of epidemics. Application to SARS-Cov-2 transmission

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For a mathematical model with five compartments suitable to the features of a certain type of epidemic transmission we discuss the identification of some system parameters by means of minimization problems for functionals involving available measurements for observable compartments. The minimization problems are treated by an optimal control technique with state constraints imposed by realistic considerations.

Robust and efficient solvers for linear or nonnlinear poromechanics

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In this talk we present efficient numerical schemes for linear and nonlinear Biot models [1, 2]. Nonlinear Lame coefficients and/or fluid compressibility or large deformations are considered. Furthermore, an erosion problem with a moving boundary is presented [3]. We use the L-scheme, see e.g. [6] or the Newton method for linearization, either monolithically or combined with a fixed stress type splitting [5, 4]. Additionally, the optimization of the stabilization parameter in the fixed-stress scheme will be discussed [7].

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Mathematics of inequality: in social sciences, economy and more

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A. Comparisons at the individual level.

A1. The leader property

Let Ω be a population composed of individuals having the same characteristics. We can compare them according to several numeric criteria: wealth, height, income,schooling, health, age, notoriety, etc. Thus any individual "i" may be characterized by a vector $\mathbf{x}_i$ with d components, where d is the number o measured characteristics.

Thus the individuals can be compared among themselves on each component. The tentation of making tops is licit at unidimensional level. But in multidimensional case, the order relation is not total, usually the objects are not comparable. In spite of that, people want to compare them in order to make a decision. So it goes.
Suppose that we know the probability distribution of the measured characteristics of the members of the population Ω . In probabilistic terms, it is an idealization, meaning that we know the probability $F_{\mathbf{X}}(B) := P(\mathbf{X} \in B)$ if $\mathbf{X}$ is a member of Ω and B is a Borel set from $\mathbb{R}^d$.
Extract n individuals from Ω,namely $(\mathbf{X}_j)_{1\leq j\leq n}$. We are interested in questions as:
- Which is the probability a_n that among these individuals exist one of them, namely $\mathbf{X}$ be "leader"? Leader means that $\mathbf{X} \geq \mathbf{X}_j$ for all $1 \leq j \leq n$.
- Which is the probability b_n that among these individuals exist one of them, namely $\mathbf{Z}$ be "minim"? Minim means that $\mathbf{Z} \leq \mathbf{X}_j$ for all $1 \leq j \leq n$.
- Which is the probability c_n that among these individuals exist both a leader $\mathbf{X}$ and a minim $\mathbf{Z}$?
If $\liminf a_n > 0$ we say that the distribution $F_{\mathbf{X}}$ (or that the population Ω) has the "leader property" ;if $\liminf b_n > 0$ then it has the "minimum property" and if $\liminf c_n > 0$ it has the "order property"
Open problem. *If Ω has both the minimum and the leader property, does it have the order property*?

A2. Absolutely optimal portfolios

A financiar market is a stochastic $d-$dimensional vector $\mathbf{X} \geq 0$. X_i means how much one gets after investing \$1 on the $i'th$ asset. A portfolio of volume S is a vector $\mathbf{s} = (s_j)_{1\leq j\leq d}$ of nonnegative numbers, interpreted as money and its value is the scalar product $\xi = \mathbf{s}'\mathbf{X}$
Question. *Which is the best portfolio of the sum S*?
This time we have to compare random variables, not vectors, as before. Among two random variables X and Y which of them is "better"?
In the frame of expected utility, better from the point of view of some utility u (i.e. increasing function) means that $\mathrm{E}u(X) \leq \mathrm{E}u(Y)$. Thus, from this point of view, if $\mathbf{s}$ and $\mathbf{t}$ are two portfolios of the same size, $\mathbf{t}$ is better than $\mathbf{s}$ if $\mathrm{E}u(\mathbf{s}'\mathbf{X}) \leq \mathrm{E}u(\mathbf{t}'\mathbf{X})$. As it is agreed that the investors are risk avoiding, then $\mathbf{t}$ is absolutely better than $\mathbf{s}$ if $\mathrm{E}u(\mathbf{s}'\mathbf{X}) \leq \mathrm{E}u(\mathbf{t}'\mathbf{X})$ for all concave utilities.
We study the problem of the existence of the absolutely optimum portfolio.We solve it completely if $d = 2$.
Open problem*: how to construct financiary markets with the property of absolutely optimal portfolios for $d \geq 3$?*

B. Comparisons at population level.

We concentrate on the following unidimensional problem: the inequality among the members of the same population having the unidimensional distribution F. It is measured by the Lorenz curve.$L_F(x) = \frac{\int_0^x F^{-1}(t)dt}{\int_0^1 F^{-1}(t)dt}$. The population F is more egalitarian than G if $L_F \geq L_G$.
This order relation is of course not a total one and people want to make tops of inequality among states. That is why more one dimensional relations have been considered:
- the Gini coefficient $\gamma(F) = 1 - 2\int_0^1 L_F(x)\,dx$
- the 20:20 ratio : $k(F) = \frac{1-F(0.8)}{F(0.2)}$
- the 10:10 ratio: $K(F) = \frac{1-F(0.9)}{F(0.1)}$
We study the relations among them.

1. Partial Differential Equations

Unstable longitudinal waves in rods with variable cross-section

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A system of differential equations describing the propagation of unsteady longitudinal waves in rods of variable cross-section is considered. The following variational problem

$$\delta J + \delta A = 0.$$

is the source of the equations and the natural boundary conditions accompanying them. Here δA is work of external force, $J = \int_{t_1}^{t_2} \int_0^l \Phi \, dx \, dt$. The function Φ is an integrand in the equation $T - \Pi = \int_0^l \Phi \, dx$. Here T, Π are kinetic and potential energies. A concrete form of the function Φ can be given only after we agree on the displacement model.
In this work, we take into account both longitudinal displacements of the section as a whole, and the transverse displacements of the points of the variable section $S(x)$ according to the following model:

$$u_1 = u(x,t), \;\; u_2 = \frac{y}{a(x)} \cdot w(x,t), \;\; u_3 = \frac{z}{a(x)} \cdot w(x,t); \quad a(x) = \sqrt{\frac{I_0(x)}{S(x)}}.$$

Here $I_0(x) = \int_{S(x)} (y^2 + z^2) \, dS$ is polar moment of inertia of the current section. So, we have two unknown functions: $u(x,t)$, $w(x,t)$. Using this model, we can define deformations and stresses, then the energies T, Π, and, finally, the form of the Φ function:

$$\Phi = \Phi\left(w, u'_x, u'_t, w'_x, w'_t\right) = \Phi\left(w, p, q, r, s\right) =$$

$$= \left\{ \frac{1}{2}\rho q^2 + \frac{1}{2}\rho s^2 - \left(\mu + \frac{\lambda}{2}\right) p^2 - 2\lambda \frac{w}{a(x)} p - (2\mu + 2\lambda) \frac{w^2}{a^2(x)} - \right.$$

$$\left. - \frac{\mu}{2a^2(x)} \left(a(x) \cdot r - a'(x) \cdot w\right)^2 \right\} S(x).$$

Then the Euler-Ostrogradskii equations with associated natural boundary conditions provide a formulation of the boundary value problem. It is solved by the method of characteristics. The following features are established: 1) even at $w \equiv 0$, the wave amplitude increases with propagation towards a decrease in the cross section, which agrees with the obvious solution of the corresponding dispersion equation; 2) in addition to this, if $w \neq 0$, an enhanced distortion of the wave profile and a slowdown in the propagation velocity are observed.

Determination of some solutions of the 2D Navier-Stokes equations

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The following system of partial differential equations are examined:

$$\begin{cases} \frac{P_x}{\mu} + uu_x + vu_y = a \cdot \triangle u + F_x \\ \frac{P_y}{\mu} + uv_x + vv_y = a \cdot \triangle v + F_y \\ u_x + v_y = 0 \end{cases} \quad (1)$$

$P = P(x,y);\ u = u(x,y);\ v = v(x,y);\ F = F(x,y);\ u_x = \frac{\partial u}{\partial x};$
$\triangle u = u_{xx} + u_{yy};\quad x, y \in \mathbb{R}.$
The system (1) describes the process of plane stationary flow of a liquid or gas. This system represents the Navier-Stokes equations in the case of 2D stationary motion of a viscous incompressible fluid. The P function represents the pressure of the liquid, and u, v functions represent the flow of the liquid or gas, F represents the external forces. The constants $a > 0$ and $\mu > 0$ is a determined parameter of the studied liquid's (of the gas) viscosity and density. We mention here that $a = \dfrac{c_0}{R_e}$, $c_0 > 0$, where R_e is the Reynolds number.
The aim of this research is to find out the exact solutions for system (1).
If in the connected area D , the functions u, v, P and F admits the bounded derivatives up to and including order 2 then the next theorem take place:

Theorem. *Let's consider that functions $\varphi(x;y)$ and $z(x;y)$ admit in D area continuous partial derivatives bounded up to including order 2 and $F(\varphi)$ is a function of the second order derivative of its argument. These functions verify the following equations (2) and (3):*

$$(\varphi_x^2 + \varphi_y^2) \cdot F'' + \triangle\varphi \cdot F' + \triangle z = \varphi \ (2), \quad \varphi_y \cdot z_x - \varphi_x \cdot z_y + a \cdot \triangle\varphi = 0 \ (3).$$

Then solutions of system (1) are determined as follows: firstly, we identify u and v from the system:

$$\begin{cases} u = \varphi_y \cdot F' + z_y, \\ v = -\varphi_x \cdot F' - z_x, \end{cases} \quad (4)$$

the function G from

$$\begin{cases} G_x = a\varphi_y - v \cdot \varphi, \\ G_y = -a\varphi_x + u \cdot \varphi, \end{cases} \quad (5)$$

and the function P from

$$G = \tfrac{1}{\mu}P - F + 0,5(u^2 + v^2) \quad (6)$$

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Coefficients inverse problem for a parabolic equation with strong power degeneration

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In a domain $Q_T = \{(x,t) : 0 < x < h, 0 < t < T\}$ we consider the coefficient inverse problem for determination of two time dependent functions $b_1 = b_1(t), b_2 = b_2(t)$ in the minor coefficient in the one-dimensional degenerate parabolic equation

$$u_t = a(t)t^{\beta}u_{xx} + (b_1(t)x + b_2(t))u_x + c(x,t)u + f(x,t) \tag{1}$$

with initial condition

$$u(x,0) = \varphi(x), \quad x \in [0,h], \tag{2}$$

boundary conditions

$$u(0,t) = \mu_1(t),\ u(h,t) = \mu_2(t),\ t \in [0,T] \tag{3}$$

and heat moments as overdetermination conditions

$$\int_0^h u(x,t)dx = \mu_3(t), \quad t \in [0,T], \tag{4}$$

$$\int_0^h xu(x,t)dx = \mu_4(t), \quad t \in [0,T]. \tag{5}$$

It is known that $a(t) > 0, t \in [0,T]$. The degeneration of the equation is caused by the power function with respect to the time variable t^{β}. The case of strong degeneration $\beta \geq 1$ is investigated. Applying the Schauder fixed point theorem there is established conditions of existence of the classical solution to the named problem. For this aim the Green functions of the boundary value problems for the heat equation and their properties are used.
We prove the uniqueness of the solution taking into account the properties of the solutions of the homogeneous integral Volterra equations of the second kind with integrable kernels.

2. ODEs and Dynamical Systems

Some variational inclusions for a Caputo-Fabrizio fractional differential inclusion

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Recently, a new fractional order derivative with regular kernel has been introduced by Caputo and Fabrizio. This new definition is able to describe better heterogeneousness, systems with different scales with memory effects, the wave movement on surface of shallow water, the heat transfer model, mass-spring-damper model etc.. Another good property of this new definition is that using Laplace transform of the fractional derivative the fractional differential equation turns into a classical differential equation of integer order.
In Control Theory, mainly, if we want to obtain necessary optimality conditions, it is essential to have several "differentiability" properties of solutions with respect to initial conditions. One of the most powerful result in the theory of differential equations, the classical Bendixson-Picard-Lindelőf theorem states that the maximal flow of a differential equation is differentiable with respect to initial conditions and its derivatives verify the variational equation.
We are concerned with fractional differential inclusions of the form

$$D^{\sigma}_{CF}x(t) \in F(t, x(t)) \quad a.e.\ ([0, T]), \quad x(0) = x_0, \quad x'(0) = x_1 \tag{1}$$

where $\alpha \in (0, 1)$, $\sigma = \alpha+1$, D^{σ}_{CF} is the Caputo-Fabrizio fractional derivative, $F : [0, T]\times\mathbf{R} \to \mathcal{P}(\mathbf{R})$ is a set-valued map and $x_0, x_1 \in \mathbf{R}$. Several results concerning the differentiability of solutions of problem (1) with respect to initial conditions are obtained.

Boundary value problem modeling for linear differential-difference equations of neutral type

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Conditions for the existence of boundary value problem solutions for various classes of differential-difference equations were considered in the papers [1, 2, 3, 4]. Sufficient conditions for the boundary value problem solution existence for integro-differential equations of neutral type were investigated in [5].
We consider the following boundary value problem

$$y''(x) = \sum_{i=0}^{n}\Bigg(a_i(x)\, y(x - \tau_i(x)) + b_i(x)\, y'(x - \tau_i(x)) \tag{1}$$

$$+c_i(x)\, y''(x - \tau_i(x))\Bigg) + f(x),$$

$$y^{(p)}(x) = \varphi^{(p)}(x),\ p = 0, 1, 2,\ x \in [a^*; a],\ y(b) = \gamma, \tag{2}$$

where $\tau_0(x) = 0$ and $\tau_i(x)$, $i = \overline{1,n}$ are continuous nonnegative functions defined on $[a,b]$, $\varphi(x)$ is a twice continuously differentiable function given on $[a^*;a]$, $\gamma \in R$,

$$a^* = \min_{0<i\leq n} \left\{ \inf_{x\in[a;b]} (x - \tau_i(x)) \right\}.$$

In this paper we investigate a scheme of boundary value problem modeling for linear differential-difference equations of neutral type with many variable deviations of the argument. A functional space is defined to which the solution of the considered boundary value problem belongs, the properties of the solution smoothness are investigated depending on the structure of the argument deviations. Simple and verifiable sufficient conditions for the boundary value problem solution existence are given.
For finding the solution of the boundary value problem an iterative computational scheme based on the spline approximation method is described. In order to take into account possible discontinuities of the boundary value problem solution derivatives, cubic splines of defect two are used for neutral type equations. Coefficient conditions for the initial equation which ensure the convergence of the iterative process are obtained. An estimate of the approximate solution error is conducted.

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On the approximation of linear systems with delay and their stability

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In the present paper, we consider applications of the approximation schemes for differential-difference equations to approximate finding nonasymptotic roots of quasipolynomials and analysis of stability for solutions of system of linear differential equations with delay. Considered a linear system of differential equations with many delays

$$\frac{dx(t)}{dt} = \sum_{i=0}^{k} A_i x(t - \tau_i), \tag{1}$$

where $x \in R^n$, A_i, $i = \overline{1,k} - n \times n$ are constant matrices, $0 = \tau_0 < \tau_1 < ... < \tau_k = \tau$.
In accordance with the scheme [1-3], we associate with equation (1) the system of ordinary differential equations

$$\frac{dz_0(t)}{dt} = \sum_{i=0}^{k} A_i z_{l_j}(t),\ l_i = [\frac{\tau_i m}{\tau}],$$

$$\frac{dz_i(t)}{dt} = \mu[z_{i-1}(t) - z_i(t)],\ i = \overline{1,m},\ \mu = \frac{m}{\tau},\ m \in N. \tag{2}$$

The following theorem is important for constructing algorithms for studying the stability of system (1).

Theorem. *If null solution of equation (1) is exponentially stable (not stable) then there exists $m_0 > 0$ such that for all $m > m_0$, null solution of system (3) is exponentially stable (not stable). If for all $m > m_0$ null solution of approximation system (3) is exponentially stable (not stable) then nul solution of equation (1) is exponentially stable (not stable).*

Using the above theorem, we can obtain an effective algorithm for stability analysis of the system

$$\frac{dx(t)}{dt} = Ax(t) + Bx(t - \tau), \tag{3}$$

where $x \in R^n$, $A, B - n \times n$ are fixed matrices, $\tau > 0$.
When evaluating zeros of the characteristic equation of the approximating system of ordinary differential equations for (3) with different values of τ remaining stability of zero solution of the approximating system, we find the delay domain τ , making system (3) to be exponentially stable.

$$A = \begin{pmatrix} -0,9 & -6,5 \\ 4,8 & -0,9 \end{pmatrix}, \quad B = \begin{pmatrix} -1,39 & -0,65 \\ 0,48 & -1,39 \end{pmatrix}$$

is asymptotically stable at $m = 500$ in system (2) if where $\tau \in (0, \tau_1) \cup (\tau_2, \tau_3)$ where $\tau_1 = 0,2862, \tau_2 = 0,8330, \tau_1 = 1,2290$.

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Integrability conditions for a cubic system with two invariant straight lines

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We consider the cubic system of differential equations having a singular point M with pure imaginary eigenvalues and two parallel invariant straight lines l_1, l_2, where $M \notin l_1 \cup l_2$. Then by using a nondegenerate transformation of variables and a time rescaling the system can be brought to the form

$$\dot{x} = y + cxy + mx^2y, \ \dot{y} = -x - gx^2 - dxy - by^2 - sx^3 - qx^2y - nxy^2 - ly^3, \tag{1}$$

where the variables and coefficients are real. For system (1) the invariant straight lines l_1, l_2 are parallel to the axis of ordinates and the singular point M which is a center or a focus is placed at the origin. The problem of distinguishing between a center and a focus is open for cubic systems. The problem of the center was solved for cubic system (1) with: four and three invariant straight lines [1]; two invariant straight lines and one irreducible invariant conic [1]; two invariant straight lines and one irreducible invariant cubic [2]. Center conditions were obtained for system (1) when $m = l = 0$ or $m = n = 0$ in [3]; when $l = 0$ in [4] and for a nine-parameter cubic system (1) that can be reduced to a Liénard type system in [5].
In this talk we consider the problem of integrability for system (1) when $lm(c^2 - 4m) \neq 0$. We prove that the cubic system (1) with a center is always Darboux integrable.

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Analysis of on an epidemic model

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In the process of understanding the mechanism of transmission and control of epidemic diseases, mathematical models have a special importance. In literature, numerous classical epidemiological models, such as SIR and SIRS models, where S, I, and R denote the number of susceptible, infectious, and recovered individuals, have been proposed and studied in detail. In order to describe a disease as accurately as possible , the incubation period of the disease must be taken into account. Therefore, the SEIR epidemic models, where E denotes the number of individuals who are infected but not yet infectious, are developed to investigate the role of the incubation period in disease transmission.
In this paper, an SEIR epidemic model with generalized nonlinear incidence and infection-dependent removal rate is going to be studied. Both the existence of the associated equilibria and their stability are analysed. The basic reproductive number is going to be calculated, deducting that this number does not depend on the functional form of the incidence, but, on the removal rate.

Keywords: epidemic models; non-linear incidence; removal rate; basic reproductive number .

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Classical dynamical systems versus quantum systems

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Some general aspects regarding classical dynamical systems and quantum systems are presented. In the classic case, the continuity equation for the probability density and the Liouville theorem about the evolution of volumes in the phase space are presented. For the quantum description of the systems in the states space are presented the Schrödinger equation on the basis of which the probability density of locating the systems in the state space is built. In order to benefit from the formalism of quantum physics, the Liouville operator is identified in the classical case, whose eigenvectors can determine a base in the phase space similar to the basic states of a Hermitian operator from quantum physics. The existing results in the literature are systematized in an original formulation.

On the analysis of the controlled Chua dynamical system in a cubic modified version

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The present paper continues some recent work on the analytical approach of the Chua dynamical system. The cubic modified version is taken into account and a simple linear controller is considered on one component of the model. The influence of parameters is analyzed and it is assessed the hypothesis when the system is driven to a stable state using the Lyapunov function method. The results will be further used to complete the panel analysis on the chaotic behavior of the model.

AMS Subject Classification (2010): 93C40, 93C15, 93D15, 93B18
Key words: adaptive control; stabilization by feedback; Chua system; linearization.

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Analyzing the Jacobi stability of Lü's circuit system

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In this paper we will made a study of the Jacobi stability of Lü's circuit system using the geometric tools of the Kosambi-Cartan-Chern theory. In order to find the Jacobi stability conditions, we will determine all five invariants of the KCC-theory which express the intrinsic geometric properties of the system, including the deviation curvature tensor which determine the Jacobi stability of the system near equilibrium points.

AMS Subject Classification (2020): 34D20, 37C20, 37C75, 53E10
Key words: Lü circuit system, Jacobi stability, KCC-theory

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Stability conditions of unperturbed motion governed by critical three-dimensional differential system of Darboux type $s^3(1,2,3)$

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We examine the three-dimensional differential system $s^3(1,2,3)$

$$\dot{x}^j = a^j_\alpha x^\alpha + a^j_{\alpha\beta} x^\alpha x^\beta + a^j_{\alpha\beta\gamma} x^\alpha x^\beta x^\gamma \quad (j,\alpha,\beta,\gamma = \overline{1,3}), \tag{1}$$

where $a^j_{\alpha\beta}$ and $a^j_{\alpha\beta\gamma}$ are symmetric tensors in the lower indices, by which a total convolution is carried out here. By a center-affine transformation, the system (1) can be brought to the critical Lyapunov form [1] and in the center-affine conditions $\eta = a^\alpha_{\beta\gamma} x^\beta x^\gamma x^\delta y^\mu \varepsilon_{\alpha\delta\mu} \equiv 0$ and $\eta_1 = a^\alpha_{\beta\gamma\delta} x^\beta x^\gamma x^\delta x^\mu y^\nu \varepsilon_{\alpha\mu\nu} \equiv 0$, from [2], the system (1) becomes a critical of Darboux type, of the form

$$\dot{x} = Cx, \quad \dot{y} = px + qy + rz + Cy, \quad \dot{z} = sx + my + nz + Cz, \tag{2}$$

where $C = 2(gx+hy+kz)+3(ax^2+by^2+cz^2+2dxy+2exz+2fyz)$, $a^2_1 = p$, $a^2_2 = q$, $a^2_3 = r$, $a^3_1 = s$, $a^3_2 = m$, $a^3_3 = n$, and $a,b,c,d,e,f,g,h,k,m,n,p,q,r,s$ are coefficients that take values from the fields of real numbers $\mathbb{R}$.

According to [3], a condition which assures us that the system (1) is critical, is $L_{2,3} \equiv \frac{1}{2}(\theta_1^2 - \theta_2) = nq - mr > 0$, where $\theta_1 = a^\alpha_\alpha$, $\theta_2 = a^\alpha_\beta a^\beta_\alpha$, are center-affine invariants of the system (1), from [2].

We will introduce the following notations:

$$M = g + hA_1 + kB_1, \quad N = a + bA_1^2 + cB_1^2 + 2dA_1 + 2eB_1 + 2fA_1B_1, \tag{3}$$

where

$$A_1 = (rs - np)L^{-1}_{2,3}, \quad B_1 = (mp - qs)L^{-1}_{2,3}.$$

Then, taking into account the Lyapunov Theorem [1, §32] and the expressions (3), we have

Theorem. *The stability of unperturbed motion, described by the critical system of Darboux type* $s^3(1,2,3)$ *of perturbed motion* (2)*, includes all possible cases in the following four:*

I $M \neq 0$, *then unperturbed motion is* **unstable***;*

II $M = 0,\ N < 0$, *then unperturbed motion is* **stable***;*

III $M = 0,\ N > 0$, *then unperturbed motion is* **unstable***;*

IV $M = N = 0$, *then unperturbed motion is* **stable***.*

In the last case, the unperturbed motion belongs to some continuous series of stabilized motions, and moreover this motion is asymptotically stable. The expressions M and N are given in (3).

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Fifth degree differential system with an invariant straight line of maximal multiplicity

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We consider the real polynomial system of differential equations

$$\dot{x} = P(x,y),\ \dot{y} = Q(x,y), \tag{1}$$

and the vector field $\mathbb{X} = P(x,y)\frac{\partial}{\partial x} + Q(x,y)\frac{\partial}{\partial y}$ associated to system (1).
Denote $n = \max\{deg(P), deg(Q)\}$. If $n = 5$ the system (1) in *called fifth degree differential system.*
A curve $f(x,y) = 0,\ f \in \mathbb{C}[x,y]$ is said to be an *invariant algebraic curves* of (1) if there exists a polynomial $K_f \in \mathbb{C}[x,y]$, $deg(K_f) \leq n-1$ such that the identity $\mathbb{X}(f) \equiv f(x,y)K_f(x,y)$ holds.
An invariant algebraic curve f of degree d for the vector field is said to have *algebraic multiplicity* m if m is the greatest positive integer such that the m-th power of f divides $E_d(\mathbb{X})$, where

$$E_d(\mathbb{X}) = det\begin{pmatrix} v_1 & v_2 & ... & v_k \\ \mathbb{X}(v_1) & \mathbb{X}(v_2) & ... & \mathbb{X}(v_k) \\ ... & ... & ... & ... \\ \mathbb{X}^{k-1}(v_1) & \mathbb{X}^{k-1}(v_2) & ... & \mathbb{X}^{k-1}(v_k) \end{pmatrix} \tag{2}$$

and v_1, v_2, ..., v_k is a basis of $\mathbb{C}_d[x,y]$ [2].
If $d = 1$ then $v_1 = 1$, $v_2 = x$, $v_3 = y$ and $E_1(\mathbb{X}) = P \cdot \mathbb{X}(Q) - Q \cdot \mathbb{X}(P)$.
The problem of the estimation of the number of invariant straight lines wich a polynomial differential system may have was considered in [1].

About the study of cubic systems with invariant straight lines of parallel multiplicity is studied in [3], and about cubic differential systems and quartic with a real invariant straight line of maximum multiplicity is investigated in [4] and [5]
In this work we show that in the class of fifth degree differential system with an invariant straight line of maximal multiplicity.

Theorem. *In the class of fifth degree differential system the maximal algebraic multiplicity of an affine real invariant straight line is equal to 13. Via an affine transformation of coordinates and time rescaling each fifth degree differential system with has an invariant straight line of algebraic multiplicity 13 can be written in the form:*

$$\dot{x} = x^5, \; \dot{y} = 1 + 5x^4 y. \tag{3}$$

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The multiplicity of the invariant straight line at the infinity for the quintic system

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Consider the real polynomial differential system of degree n, i.e. a differential system

$$\dot{x} = P(x, y), \quad \dot{y} = Q(x, y), \tag{1}$$

where $P, Q \in \mathbb{R}[x, y]$, $\max\{\deg P, \deg Q\} = n$ and $GCD\,(P, Q) = 1$.
A straight line $Ax + By + C = 0$, $A, B, C \in \mathbb{C}$, $A^2 + B^2 \neq 0$, is called invariant for the system (1) if there exists a polynomial $K(x, y)$ such that the identity $AP(x, y) + BQ(x, y) \equiv (Ax + By + C)K(x, y)$ holds.
According to [1], if the system (1) has sufficiently many invariant straight lines considered with their multiplicities, then we can obtain a Darboux first integral for it. There are different types of multiplicities of these invariant straight lines, for example: parallel multiplicity, geometric multiplicity, algebraic multiplicity, etc [2]. In this work we will use the notion of algebraic multiplicity

of an invariant straight line
Let's denote by l_∞ the invariant straight line of the system (1) situated at the infinity. Every polynomial differential system has such an invariant straight line and its multiplicity is at least one.
It's easy to show that if $n = 1$, i.e. the system (1) is an affine one, then maximal multiplicity of l_∞ is equal to three.
Also, it's known that for the quadratic systems, the straight line l_∞ can have multiplicity at most five. In [3] it was obtained tht the maximal multiplicity of the straight line l_∞ for the cubic systems is equal to 7. In [4,5] it was shown that for the cuartic systems the invariant straight line l_∞ has maximal multiplicity equal to ten.
In this work we show that in the class of quintic differential systems the maximal algebraic multiplicity of the line at infinity is at least 13.

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Off-label use of epidemiological modelling: how to estimate and curtail rumor spreading

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We propose and investigate a model for rumor spreading and control which accounts for different attitudes towards rumor spreading and for a distinct type of variable, related to the strength and effectiveness of rumor inhibiting mechanisms. The control mechanisms essentially amount to budgeting and to adjusting the attitude of spreaders. The existence and stability of the trivial (rumor-free) equilibrium and of the semi-trivial equilibrium are characterized in terms of two threshold parameters which quantify the influence of both categories of spreaders. The existence of a positive (rumor-prevailing) equilibrium is also established, its stability being discussed with the help of a bifurcation theorem. A nonstandard finite difference (NSFD) scheme is devised to construct approximate solutions while preserving their positivity, necessary conditions for the existence of the optimal discrete rumor spreading controls being then established.

3. Mathematical Modeling

Homogenization of a diffusion problem in thin filtering materials

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We shall present homogenization results for a diffusion problem in a thin heterogeneous medium made up of two materials separated by an interface which is imperfect. The mathematical model under consideration has applications, for instance in the study of filtering materials, such as biological tissues, textiles, or paper.

Uniqueness and continuous dependence for thermo-electro-viscoelasticity of Green-Naghdi type

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We consider a one-dimensional body in the framework of the linear theory of thermo-electro-viscoelasticity of Green-Naghdi type, which was introduced in [1]. This model presents practical applications, since in biomechanics, a tissue constituent is usually analyzed from the point of view of the uniaxial behaviour, see for example [2], [3].
The Green-Naghdi theory is based on an entropy balance law rather than an entropy inequality, see [4]. The linearized form of this theory leads to three different models of heat conduction. The Green-Naghdi linear model of type III implies the transmission of heat as thermal waves at finite speed, as compared with Fourier's law.
Our mathematical model consists of a system of integro-differential equations with initial and boundary conditions. With an appropriate form of the constitutive equations, we prove a uniqueness theorem for the solution to the mixed boundary-initial-value problem by using the Laplace transform. Finally, we present a result of continuous dependence upon the supply terms.
This is a joint work with Professor M. Marin (from Transilvania University of Braşov) and with Professor A. Montanaro (from the University of Padua, Italy).

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Using grey modeling in the analysis of COVID-19's spread in Romania

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Mathematical modeling is one of the widely used scientific tools which allow us to predict the evolution of the disease and take intervention measures accordingly.
At least four types of models can be used to study the evolution of infectious diseases: the compartmental models (deterministic differential equations like SIR or SEIR), agent-based models (that consider people as lattice sites on a network, each site being in a specific stage-susceptible, exposed, infected, etc.- that is modified according to some precise rules), stochastic differential equations (differential equations which feature random variables) and data-driven model (that simply take the existing data disease's spread over a period and use machine learning methods to generate a forecast for the next short period, paying little or no attention to the underlying processes driving the spread of the disease). Each of them has advantages and limitations.
The philosophy of the grey modeling is the use of appropriate deterministic models (dynamical systems) whose coefficients are determined using data series from measurements. It is a combination of the compartmental and data-driving techniques.
The purpose of the article is to apply the grey modeling in order to identify a mathematical model that fits as accurately as possible the statistical data on the spread of COVID 19 in Romania.
The study considers two categories of population (infected, respectively vaccinated people) about which daily statistical data are available. We study each population using the grey Verhulst model and we try to point out a correlation between them using the grey Lotka-Volterra model. Short time predictions are also presented and discussed.

A mathematical answer to the question of whether or not vegetation can mitigate surface runoffs

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To find and understand how different factors can attenuate the surface runoffs is an important and actual challenge in the context of the present-day severe flooding led by the heavy rainfalls which start appearing more often lately due to the global climate changes. Here, we introduce a mathematical model that takes into account the presence of vegetation on the soil surface and the water-soil and water-plant interactions. We qualitatively and quantitatively estimate the role of the plants in the continuum soil-plant-atmosphere by comparing measured results of different tests found in the literature with the one given by our model.

Modelling and existence results in the Cosserat shell theory

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We present a new geometrically nonlinear Cosserat shell model incorporating effects up to order $O(h^5)$ in the shell thickness h. The method that we follow is an educated 8-parameter ansatz for the three-dimensional elastic shell deformation with attendant analytical thickness integration, which leads us to obtain completely two-dimensional sets of equations in variational form. We give an explicit form of the curvature energy in terms of the wryness tensor. Moreover, we consider the matrix representation of all tensors in the derivation of the variational formulation, because this is convenient when the problem of existence is considered, and it is also preferential for numerical simulations. The step by step construction allows us to give a transparent approximation of the three-dimensional parental problem. An existence of the solution is obtained for the proposed nonlinear model.

Missing data in the oil industry. Method of imputations and the impact on reserve estimation

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The missing data can be a major challenge for any researcher who needs the accuracy and reliability of his statistical studies. In any statistical study, missing data, regardless of number, can cause a dramatic shrink of sample size and, because of that, a nonreliable statistical conclusion. Due to this problem, the accuracy of any statistical method applied, such as estimates of population parameters, and statistical applications will be impaired and statistical power will be weaker. All researchers will encounter this problem eventually, during the empirical research process, and it will be up to them to evaluate how they will address the missing data. The missing data can be caused by many different reasons; they are classified as missing completely at random (MCAR), missing at random (MAR), and missing not at random (MNAR). In the oil industry, in the process of oil and gas exploration or production, the missing data can be caused due to different reasons and they can be of any type of classification mentioned before. As a result of missing data from an oilfield, the matrix of geological data consisting of parameters values such as density, permeability, oil saturation, temperature, pressure, etc., will affect the oilfield geological estimates, which will be reflected in the oilfield reserves estimation, the oilfield production performance evaluation, etc. The assessment of the type of missing data is very important as well as finding and implementing the best method of the imputation of filling in the missing data. We will analyze and apply several methods of missing data imputation and evaluate the impact on reserves estimation of the M/D oilfield in Albania.

Keywords: missing, data, oilfield, reserves, geology, imputation, methods

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Estimation of brachial blood pressure for individuals aged 25 years with multiple linear regression

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The flowrate, velocity and pressure changes are important parameters in the hemodynamic analysis of blood flow in the vessels.The pressure variation during blood flow plays an important role in hemodynamic studies. In CFD studies the pressure, flowrate and velocity inlet or outlet boundary conditions are effective for precise and accurate analysis. Specifying the pressure wave obtained from real patient datas as an inlet or outlet condition during the CFD analysis ensures that the analysis is close to reality. For this reason, blood pressure waves can be obtained using mathematical models, and very close to real blood pressure waves can be estimated with Machine Learning algorithms.
In this study, the blood pressure wave was tried to be estimated by taking various characteristics of 25-year-old individuals. The available data was analyzed first and it was determined which parameters played an active role. Then % 70 of this data was chosen as training data and the rest was reserved as test data. Considering the important parameters, a multiple linear regression model was made and pressure waves were created with the existing parameters thanks to the model. With the test data, it was evaluated how successful these predictions were. Virtual pressure waves required for other studies could be obtained from the model that was used in this study.

Keywords: Blood pressure, Machine Learning, Multiple linear regression.

Use of adequate mathematical descriptions of dynamical systems

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The article discusses the possibility of using adequate mathematical descriptions in order to predict the motion of dynamic systems by methods of mathematical modeling. All questions are investigated in a deterministic setting. Two main criteria for the adequacy of dynamic systems are proposed: quantitative and qualitative [1]. The fulfillment of these criteria of adequacy makes it possible to increase the forecasting accuracy and the efficiency of motion control of dynamical systems.

Introduction. For simplicity, we consider dynamical systems of the following type:

$$\dot{x}(t) = Cx(t) + Dz(t), \tag{1}$$

where C, D - matrices with constant coefficients which are given approximately,
$x(t) = (x_1(t), x_2(t), ..., x_n(t))^T \in X$ - vector-function variables, characterized the state of process (T - a mark of transposition), $z(t) = (z_1(t), z_2(t), ..., z_k(t) \in Z$ - vector-function of external load;X, Z are normalized functional spaces.

Let us give several definitions for convenience. *A mathematical model* of a real object will be called mathematical dependencies and connections between the elements of a mathematical model. These elements are chosen on the basis of the interests of the researcher himself and the ultimate goals of study of the object. The functions of external influences and external loads that are present in the mathematical model of the object in the form of symbols will be called *models of external loads.* The initial conditions, boundary conditions, and other conditions for the mathematical model will be called *additional conditions.*

The totality of the mathematical model of the object, models of external influences and additional conditions will be called *a mathematical description* of the object. The study of the behavior of the mathematical model of an object under the influence of models of external loads and additional conditions will be called *mathematical modeling or mathematical simulation.*

Currently, there are two main approaches to the problem of constructing of mathematical description [1],[2]:

- for a mathematical model with a priori chosen structure and inaccurate parameters, a model of external load is determined, which together with the mathematical model of the process and additional conditions provide the good coincidence with experiment;
- a models of external loads and additional conditions are given a priori and then parameters of mathematical model or of its structure are selected, such way that results of mathematical simulation match up good with experiment.

The most important application of the results of mathematical modeling is to predict the behavior of a dynamical system. For this purpose, it is necessary to have a good mathematical model of the system, a reliable model of the external load and the correct additional conditions. However, the existing approaches to constructing mathematical descriptions do not guarantee the fulfillment of these requirements. The problem lies in the ambiguity of the requirement for "good agreement with experiment". Subjective factors have a great influence here. Because of this, the results of mathematical modeling cannot be reliable in predicting the behavior of a dynamical system.

The concept of an adequate mathematical description was proposed to overcome this obstacle.

General definition: A mathematical description will be called *an adequate mathematical description (AMD) of quantitative type* of the process under study if the results of mathematical

simulation using this description coincide with experimental data with the accuracy of experimental measurements [1].
It should be noted that the components of an adequate mathematical description may not correspond to the physical concepts of dynamical systems [1]. Obviously, there are an infinite number of such descriptions for any approach to synthesizing a mathematical description.
We will consider only the possibilities of the first approach.
Let the selected structure of the mathematical model of the physical process include parameters $p = (p_1, p_2, ..., p_k)^T$ which are reflecting the actual physical characteristics. The structure of the mathematical model also includes dependencies that reflected real physical patterns and dependencies of the process under study. In addition, it is necessary to require that the interconnections between the parameters of a mathematical model comply with the physical laws of the process being studied and the main external loads had been included. This important correspondence will be called *the main correspondence* (MC). The execution of the MC will be believed the fulfillment of the criterion of *adequacy of the qualitative type*.
Adequate mathematical descriptions of dynamic systems make it possible to predict motion if two criteria of adequacy are met.
Possible Algorithm of Forecast Estimation with Guaranty. Let the prognosis of the change in maximum deviation from zero of one variable $x_k(t)$ of the dynamical system (1) be carried out. It will also assume that system (1) satisfies the criterion of the adequacy of the qualitative type (criterion MS). The algorithm for the synthesis of an adequate mathematical description for the purposes of variable $x_k(t)$ forecasting motion of a dynamic system (1) is proposed:
- the possible or desired spread of the parameters $p \in D$ of the mathematical model of the dynamic system is determined for future conditions, the external load is not changed.
- for the given experimental measurement, adequate mathematical descriptions are synthesized for each variant of possible changes in the parameters of the mathematical model p of the dynamic system in the future.
- the value $\hat{x}_k$ of the deviation state variable $x_k(t)$ of interest for the time interval $[0, T]$ is being investigated for each variant of possible changes in the parameters p of the mathematical model of the dynamic system, for example, the maximum deviation of the variable $x_k(t)$ from zero on $[0, T]$

$$\hat{x}_k = F[x_k(t)] = \|x_k\|_{C[0,T]}.$$

- then among the received $\hat{x}_k$ the greatest $\hat{x}_{p^{max}}$ is defined.
- further to obtain a guaranteed forecast of the motion of the dynamic system with the indicated purpose, mathematical simulation using an adequate mathematical description with a vector of the parameter p^{max} is executed.
The results of mathematical simulation give the maximum deviation of the variable state $\hat{x}_k$ from zero for any vector $p \in D$ i. e. with guarantee.
Some other possibilities of using adequate mathematical descriptions are presented in [1].

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A solution for assessment of Air Quality Index (AQI) using auto-regressive models combined with fuzzy logic techniques

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Forecasting and analysis of the Air Quality Index (AQI) is a subject of high interest for public health. In this work, we present a method for predicting the air quality index based on the interval decomposition of the range of values obtained by measuring the elements that form the AQI, the association of numerical values to these intervals, processing with auto-regressive methods of time series obtained and value prediction, and determination of anticipated AQI with hierarchical fuzzy inference system (HFIS). The HFIS allows a classification process that is more efficient with an easier interpretation of the results. This type of algorithm can improve the current decision support tools to assist in a timely manner the local authorities to adopt proper plans that minimize the resulting ecological and epidemiological impacts, especially in a smart city.

Scaling effects on the periodic homogenization of a reaction-diffusion-convection problem posed in homogeneous domains connected by a thin composite layer

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We study the question of periodic homogenization of a variably scaled reaction-diffusion problem with non-linear drift posed for a domain crossed by a flat composite thin layer. The structure of the non-linearity in the drift was obtained in earlier works as hydrodynamic limit of a TASEP process for a population of interacting particles crossing a domain with obstacle. Using energy-type estimates as well as concepts like thin-layer convergence and two-scale convergence, we derive the homogenized evolution equation and the corresponding effective model parameters for a regularized problem. Special attention is paid to the derivation of the effective transmission conditions across the separating limit interface in essentially two different situations: (i) finitely thin layer and (ii) infinitely thin layer.

This study should be seen as a preliminary step needed for the investigation of averaging fast non-linear drifts across material interfaces – a topic with direct applications in the design of thin composite materials meant to be impenetrable to high-velocity impacts.

This is joint work with I. de Bonis (Benevento, Italy), E.N.M.Cirillo (Rome, Italy), and A. Muntean (Karlstad, Sweden

Mathematical modeling of the aeroelastic system "pipeline - pressure sensor"

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The initial boundary value problem corresponding to the model of a mechanical system "pipeline-pressure sensor" is considered

$$\varphi_{tt} = a_0^2(\varphi_{xx} + \varphi_{yy}), \quad x \in (0, l), \quad y \in (0, h), \tag{1}$$

$$\varphi_y(x, 0, t) = \varphi_y(x, h, t) = 0, \quad x \in (0, l), \tag{2}$$

$$\varphi_x(l, y, t) = w_t(y, t), \quad y \in (0, h), \tag{3}$$

$$-\rho_0\varphi_t(0, y, t) = P(y, t), \quad y \in (0, h), \tag{4}$$

$$P_0 - \rho_0\varphi_t(l, y, t) - P_* = L(w(y, t)), \quad y \in (0, h). \tag{5}$$

In (1)-(5) $\varphi(x, y, t)$ is the velocity potential describing the movement of a compres-sible working medium in a pipeline with straight walls $y = 0$, $y = h$; $w(y, t)$ is the deformation of the elastic element of the sensor located at the end of the pipeline $x = l$; ρ_0, P_0, a_0 are the density, the pressure, the speed of the sound, corresponding to the state of rest of the working medium; $P(y, t)$ is the given law of pressure variation of the working medium at the inlet to the pipeline $x = 0$; P_* is the external action on an elastic element; indices x, y, t below denote partial derivatives with respect to coordinates x, y and time t. The differential (or integro-differential) operator $L(w(y, t))$ in the equation (5) can be specified in different ways depending on the selected model of a rigid deformable body.

The system of equations (1)-(5) is supplemented with the initial conditions for the functions $\varphi(x, y, t)$ $w(y, t)$, as well as boundary conditions for $w(y, t)$ at $y = 0$, $y = h$ corresponding to the type of fixing of the elastic element.

Several methods have been developed for solving the problem, which are based on the methods: finite differences, Galerkin, averaging. The method is also presented that leads to the study of a differential equation with a deviating argument. Comparison of the results obtained by different methods is carried out.

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4. Real, Complex, Functional and Numerical Analysis

Optimal regulation width quadratic cost functional of a degenerate system

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In this paper we consider the regulator problem for the singular system

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}t}(My) + Ly = Bu(t), \quad 0 < t < \tau \\ &My(0) = My_0\end{aligned}$$

where L, M are closed linear operators in the real Hilbert space H, B is a continuous linear operator from the Hilbert space U into H, the domain $D(L)$ of L is contained into the domain $D(M)$ of M, L has a bounded inverse, y_0 is a given element in $D(L)$.

Comparisons between various types of approximately linear spaces

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The introduction and study of approximately linear spaces have been motivated by several applications in optimization, computer science, probabilities. The aim of this talk is to present some comparisons between various types of approximately linear spaces.

Multiple weak solutions for higher-order problems

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We are concerned with the multiplicity of a class of fourth-order problems with variable exponents and Navier boundary condition. The problem under investigation is considered over a general class of domains and it involves Leray-Lions type operators and a nonlinearity f. The relation between the variable exponents covers situations that cannot appear in the constant exponent case.

The talk is based on a joint work with Dr. Alejandro Velez-Santiago (University of Puerto Rico at Mayaguez).

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Time-frequency analysis for two-wavelet localization operators associated to the windowed linear canonical transform (WLCT)

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The main aim of this work is to introduce and study a class of bounded linear operators $L_{\sigma,\varphi,\psi}$ from suitable Lebesgue spaces $L^p(\mathbf{R})$ to themselves, $1 \leq p \leq \infty$, which are related to the windowed linear canonical transform (WLCT) G^A_φ, where $\sigma \in L^1(\mathbf{R}^2) \cup L^\infty(\mathbf{R}^2)$ in the beginning and afterwards $\sigma \in L^r(\mathbf{R}^2)$, $1 \leq r \leq \infty$, $\varphi, \psi \in L^q(\mathbf{R})$, $1 \leq q \leq \infty$ and A is a 2×2 real parameter matrix such that $\det(A) = 1$.
Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be such a unimodular real matrix (i.e. $a, b, c, d \in \mathbf{R}$ and $\det(A) = ad - bc = 1$). Then the linear canonical transform (LCT) of a signal $f \in L^2(\mathbf{R})$ with respect to parameter matrix A is defined by $L_A(f)(\omega) = \begin{cases} \int_{\mathbf{R}} K_A(x,\omega) f(x)dx, & b \neq 0 \\ \sqrt{d}\, e^{i\frac{cd}{2}\omega^2} f(d\omega), & b = 0, \end{cases}$ where $K_A : \mathbf{R}^2 \to \mathbf{C}$ is called the kernel of the LCT and is defined by

$$K_A(x,\omega) = \frac{1}{\sqrt{2\pi i b}} e^{\frac{i}{2b}(ax^2 - 2x\omega + d\omega^2)},\ b \neq 0,$$

for all $(x,\omega) \in \mathbf{R}^2$. The windowed linear canonical transform (WLCT) of a function $f \in L^1(\mathbf{R})$ (or $L^2(\mathbf{R})$) with respect to the window $\varphi \in L^\infty(\mathbf{R})$ (or $L^2(\mathbf{R})$) is defined by

$$G^A_\varphi(f)(\omega,u) = \int_{\mathbf{R}} f(x)\overline{\varphi^A_{\omega,u}(x)}dx,\ (\omega,u) \in \mathbf{R}^2,$$

where $\varphi^A_{\omega,u}(x) = \overline{K_A(x,\omega)}\varphi(x-u),\ x \in \mathbf{R},\ (\omega,u) \in \mathbf{R}^2$. Now, let $\varphi, \psi \in L^2(\mathbf{R})$ be two window functions such that $(\varphi,\psi) \neq 0$, where $(\ ,\ )$ denotes the inner product in $L^2(\mathbf{R})$.
Then the two-window (or two-wavelet) localization operator associated with the WLCT is defined on in $L^2(\mathbf{R})$ by

$$L_{\sigma,\varphi,\psi}(f)(x) = \frac{1}{(\varphi,\psi)} \int_{\mathbf{R}}\int_{\mathbf{R}} \sigma(\omega,u) G^A_\varphi(f)(\omega,u)\psi^A_{\omega,u}(x)d\omega du,$$

for all f in $L^2(\mathbf{R})$ and $x \in \mathbf{R}$, where $\sigma \in L^1(\mathbf{R}^2) \cup L^\infty(\mathbf{R}^2)$ or if we interpret the definition of $L_{\sigma,\varphi,\psi}$ in a weak sense

$$(L_{\sigma,\varphi,\psi}(f), g) = \frac{1}{(\psi,\varphi)} \int_{\mathbf{R}}\int_{\mathbf{R}} \sigma(\omega,u) G^A_\varphi(f)(\omega,u)\overline{G^A_\psi(g)(\omega,u)}d\omega du,$$

for all f and g in $L^2(\mathbf{R})$.
Our main goal in this work is to study the boundedness, compactness and Schatten-von Neumann properties of two window localization operators associated with the WLCT.

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On order continuous Banch C(K)-modules

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Let X be a Banach space and $C(K)$ be a Banach algebra of all continuous real or complex valued functions defined on a compact Hausdorff space K. We say that X is a Banach $C(K)$-module if the bilinear mapping

$$b : C(K) \times X \to X, b(a, x) = a.x$$

satisfies the following conditions:

i. $a.(b.x) = (a.b).x$ for all $a, b \in C(K)$ and $x \in X$,
ii. $m(1) = I$, where 1 is the constant one function and I is the identity operator on X.
iii. The bilinear mapping b is continuous. That is , the inequalty

$$\|a.x\| \leq \|a\|\|x\|$$

holds for all $a \in C(K)$ and $x \in X$.
In this talk, we present in which case a Banach $C(K)$-module satisfies the order continuity property.

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On a Boltzmann-like kinetic model with inelastic collisions and chemical reactions

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We consider a Boltzmann-like kinetic model describing a real gas with (possible) multiple inelastic collisions, including reactive collisions. We prove the global-in-time existence and uniqueness of solutions to the initial value problem associated to the model, in the case of collision-reaction laws compatible with a generalized form of the so-called Povzner inequality. The proof applies recent results on a class of nonlinear evolution equations in ordered Banach spaces, obtained by abstracting common order-monotonicity properties of some models of the so-called collisional kinetic theory.

Metric-preserving functions with respect to an intrinsic metric

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Metric-preserving functions have been studied in general topology from a theoretical point of view and have applications in fixed point theory, as well as in metric geometry to construct new metrics from known metrics.
Given a particular metric d_0 on a subset X_0 of the complex plane, we look for functions $f : [0, \infty) \to [0, \infty)$ that are metric-preserving with respect to (X_0, d_0), i.e. $f \circ d_0$ is also a metric on X_0. Here we consider the cases where d_0 is a hyperbolic metric, a triangular ratio metric or some Barrlund metric. These metrics and other related intrinsic metrics are recurrent in the theory of quasiconformal mappings.
We prove the subadditivity of every function $f : [0, \infty) \to [0, \infty)$ which is metric-preserving with respect to (X_0, d_0), in the following cases:

1 $X_0 = \Omega$ is a proper simply-connected subdomain of $\mathbb{C}$ and $d_0 = \rho_\Omega$ is the hyperbolic metric on Ω;

2 $X_0 = \mathbb{H}$ is the upper half-plane and $d_0 = \text{arctanh} s_{\mathbb{H}}$, where $s_{\mathbb{H}}$ is the triangular ratio metric on $\mathbb{H}$;

3 X_0 is a radial segment of the unit disk $\mathbb{D}$ and d_0 is the restriction to $X_0 \times X_0$ of $\text{arctanh} s_{\mathbb{D}}$, where $s_{\mathbb{D}}$ is the triangular ratio metric on $\mathbb{D}$. The analogous case where X_0 is a circle $|z| = r < 1$ is also discussed.

4 X_0 is a line $\text{Im}\,(z) = c > 0$ and d_0 is the restriction to $X_0 \times X_0$ of the metric $\varphi_c \circ b_{\mathbb{H},2}$, where $b_{\mathbb{H},2}$ is the Barrlund metric on the upper half-plane $\mathbb{H}$ and $\varphi_c(t) = \frac{\sqrt{2}t}{\sqrt{t^2+4c^2}}$, $t \in [0, \infty)$.

Error estimates for semi-discrete finite element approximations for a moving boundary problem capturing the penetration of diffusants into rubber

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We study a semi-discrete finite element approximation of weak solutions to a moving boundary problem that models the diffusion of solvent into rubber. We report on both a priori and a posteriori error estimates for the mass concentration of the diffusants, and respectively, for the position of the moving boundary. Our working techniques include integral and energy-based estimates for a nonlinear parabolic problem posed in a transformed fixed domain combined with a suitable use of the interpolation-trace inequality to handle the interface terms. Numerical illustrations of our FEM approximations are within the experimental range and show good agreement with our theoretical investigation.

This is joint work with Y. Wondmagegne and A. Muntean (Karlstad, Sweden)

Effect of approximation through simple functions on eigenvalues

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It is well known that eigenvalues can be seriously changed by small perturbations of the parameters in the initial problem. We consider a well-posed eigenvalue problem, depending on a continuous function as parameter. This problem originates in some studies concerning the optimization of the flow displacements in porous media (or some analogue models). We approximate the continuous function by a simple (step) function. The eigenvalue problem for the simple function has no solution. As a consequence, we show that the multi-layer method introduced in [1] (in order to minimize the Saffman-Taylor instability) makes no sense. In [2]-[3] we proved that perturbations of some parameters in similar problems can give unbounded eigenvalues for large wavenumbers.

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Isac's cones in general linear spaces

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The main aim of this research work is to present the Isac's Cones in the general context of the Linear Spaces viewed as Locally Convex Spaces thanks to the Minkowski functionals.Thus, if$A$ is a non-empty subset of an arbitrary real or complex linear space X, then the coresponding Minkowski functional $p_A : X \to [0, \infty]$ defined by $p_A(x) = \inf\{\chi > 0 : x \in \chi A\}, \forall x \in X$ is a seminorm whenever the set A is convex $(\alpha x + (1-\alpha) \in A, \forall x, y \in A, \alpha \in [0,1])$, circled(balanced) $(\alpha a \in A, \forall a \in A, \alpha \in R, |\alpha| \le 1)$ and absorbing $(\forall x_0 \in X, \exists \alpha_0 > 0 : \alpha a \in A, \forall \alpha \in R, |\alpha| \le \alpha_0)$. Consequently, $(X, \{p_A : \emptyset \ne A \subseteq X\})$ is a Hausdorff locally convex space and all the results concerning the Isac's cones can be applied in this background.

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Approximation of functions on the unit bicircle in generalized Hölder spaces

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Let $\omega(\delta_1, \delta_2)$ be a modulus of continuity; $\Omega_1(\delta)$, $\Omega_2(\delta)$ are the corresponding elementary moduli of continuity [1] satisfying Zygmund-Bari-Stechkin conditions; $\gamma = \gamma_1 \times \gamma_2$ is Cartesian product of unit bicircles. Let $H_\omega(\gamma)$ denotes the space of continuous functions $x(t, \tau)$ (on γ) whose moduli of continuity satisfy the next conditions:

$$\omega(x; \delta_1, \delta_2) \le c_1 \omega(\delta_1, \delta_2);$$

$$\omega_{1,1}(x; \delta_1, \delta_2) \le c_2 \Omega_1(\delta_1) \Omega_2(\delta_2)$$

(where $\omega_{1,1}(x; \delta_1, \delta_2)$ is mixed modulus of continuity of the second order). Introduce a norm in this space in the following way:

$$\|x\|_{H_\omega} = \max_{(t,\tau)\in\gamma} |x(t,\tau)| + \sup_{\delta_1^2+\delta_2^2 \ne 0} \frac{\omega(x; \delta_1, \delta_2)}{\omega(\delta_1, \delta_2)} + \sup_{\delta_1^2+\delta_2^2 \ne 0} \frac{\omega_{1,1}(x; \delta_1, \delta_2)}{\Omega_1(\delta_1)\Omega_2(\delta_2)} =$$

$$= \|x\|_C + H(x;\omega) + H^{t\tau}(x;\omega).$$

$H_\omega(\gamma)$ is Banach space for this norm.
Let $(L_{mn}x)(t;\tau)$ denotes an interpolational Lagrange polynomial of the function $x(t;\tau)$ with respect to the system of equidistant points. Let operator Φ_{mn} assignes to any function $x(t;\tau) \in H_\omega$ the mn-th partial sum ot its Fourier series.
The following results are established

Theorem 1. *Let $x(t;\tau) \in H_\omega$. Then*

$$\|x - L_{mn}x\|_C \leq c \ln m \ln n H(x;\omega)\, \omega\left(\frac{1}{m}, \frac{1}{n}\right),$$

$$\|x - \Phi_{mn}x\|_C \leq (1 + c\ln(2m+1)\ln(2n+1))\, H(x;\omega)\, \omega\left(\frac{1}{m}, \frac{1}{n}\right).$$

Theorem 2. *Let $x(t;\tau) \in H_{\omega^{(1)}}$; $\omega^{(2)}(\delta_1, \delta_2)$ is such that $H_{\omega^{(1)}} \subset H_{\omega^{(2)}}$ and $\Omega_1^{(1)}(\delta)/\Omega_1^{(2)}(\delta)$, $\Omega_2^{(1)}(\delta)/\Omega_2^{(2)}(\delta)$ are increasing functions. Then*

$$\|x - L_{mn}x\|_{H_{\omega^{(2)}}} \leq c \ln m \ln n \frac{\omega^{(1)}\left(\frac{1}{m}, \frac{1}{n}\right)}{\Omega_1^{(2)}\left(\frac{1}{m}\right)\Omega_2^{(2)}\left(\frac{1}{n}\right)} \|x\|_{H_{\omega^{(1)}}}, \tag{1}$$

$$\|x - \Phi_{mn}x\|_{H_{\omega^{(2)}}} \leq c \ln m \ln n \frac{\omega^{(1)}\left(\frac{1}{m}, \frac{1}{n}\right)}{\Omega_1^{(2)}\left(\frac{1}{m}\right)\Omega_2^{(2)}\left(\frac{1}{n}\right)} \|x\|_{H_{\omega^{(1)}}}.$$

Let's now consider the space L_p, $p > 1$.

Theorem 3. *Let $x(t;\tau) \in H_\omega$. Then*

$$\|x - L_{mn}x\|_{L_p} \leq (1 + c(p))\, H(x;\omega)\, \omega\left(\frac{1}{m}, \frac{1}{n}\right).$$

Remark 1. *If under the conditions of Theorems 1–3 the partial derivatives of the function $x(t;\tau)$ of orders p and q with respect to t and τ belong to the space H_ω ($x(t;\tau) \in H_\omega^{(p,q)}$), that the estimations in the mentioned theorems can be considerably improved. For example, formula (1) will look like this:*

$$\|x - L_{mn}x\|_{H_{\omega^{(2)}}} \leq c \ln m \ln n \left(\frac{1}{m^p} + \frac{1}{n^q}\right) \frac{\omega^{(1)}\left(\frac{1}{m}, \frac{1}{n}\right)}{\Omega_1^{(2)}\left(\frac{1}{m}\right)\Omega_2^{(2)}\left(\frac{1}{n}\right)} \|x\|_{H_{\omega^{(1)}}}.$$

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Vector integrals with applications in mathematical economics

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Given that commodity space in a mathematical model of an economy is often a vector space, we are interested to find suitable aggregate operators. In this paper such operators will be presented through vector integrals . Some properties as the "core-Walras equivalence" results are given.

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5. Probability Theory, Mathematical Statistics, Operations Research

On a generalization of reliability measures for semi-Markov repairable systems

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This work is dealing with reliability modelling for repairable systems with time dependence. We propose a generalization of reliability indexes such as reliability, interval reliability and availability for homogeneous semi-Markov repairable systems in discrete time. The proposed measure, called sequential interval reliability, is the probability that the system works in a sequence of non-overlapping time intervals. The necessary framework for calculating that measure according with the asymptotic theory is provided. Finally, numerical results are placed and a brief discussion on the possible applications in real world scenarios is presented.

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Neural network prediction intervals for composite regression models

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A composite distribution is a two-component spliced distribution, which generally equals a lighter-tailed probability density function (pdf) up to a threshold and a heavy-tailed pdf thereafter. Such distributions are used to model heavy-tailed actuarial loss data characterized by a large number of claims with small size and few claims with large size. In this study, we consider the Lognormal-Pareto composite regression model obtained by introducing covariates in the Lognormal-Pareto composite distribution. The statistical inference for such a model is very challenging due to the unknown threshold where the composite distribution changes its shape. Therefore, we approach the prediction problem by means of neural networks with the purpose to obtain a prediction interval for the expected value of this regression model; this expected value is the basis of the insurance premium. We discuss the results of this technique on some generated data sets.

h-transform for Bochner subordinate L^p-semigroups

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We show that the subordination induced by a convolution semigroup (subordination in the sense of Bochner) of a C_0-semigroup of sub-Markovian operators on an L^p space is actually associated to the subordination of a right (Markov) process. As a consequence, we solve the martingale problem associate with the L^p-infinitesimal generator of the subordinate semigroup. It turns out that an enlargement of the base space is necessary. A main step in the proof is the preservation under such a subordination of the property of a Markov process to be a Borel right process. We also investigate the h-transform of a subordinate C_0-semigroup of sub-Markovian operators on an L^p space.
The results were obtained jointly with Lucian Beznea and Ana-Maria Boeangiu.

Method for solving the linear multicriteria problem in integers

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A wide range of practical optimization problems in various fields lead to the solution of multicriteria linear optimization models in integers. There is a growing increase in their importance. Into the current paper we propose a method for solving the multicriteria model of linear type in integers of interactive type. Thus, the decision maker, initially assigning a certain utility to each criterion, will finally build a uni-criterion model of linear optimization in integers. The imposition of each criterion quantified in the synthesis function remains at the discretion of the decision maker, the optimal values and weight being calculated in whole or real numbers, which does not change the optimal solution of the model. For this purpose the decision maker has at his disposal a selection of combinatorial values, which depends on the number of criteria in the initial model. When changing the value of utilities, the decision maker can determine a new optimal solution of the initial model. The theoretical justification of the algorithm is brought in the paper. The algorithm was tested on several examples, proving its veracity.

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6. Algebra, Logic, Geometry (with applications)

On geometric weight systems

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We put the Rozansky-Witten weight systems obtained from Lie algebroids by Voglaire & Xu, into the general machine provided by Konsevich in the context of foliations and formal geometry.
This is a joint work with Dorin Cheptea (IMAR).

Contact geometry of hypersurfaces of the nearly Kählerian six-sphere

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1. It is known that the six-dimensional sphere S^6 equipped with the canonical nearly Kählerian structure was the first example of non-Kählerian almost Hermitian manifold. That is why it presents a special interest for researchers in Hermitian geometry. Such outstanding mathematicians as A. Gray, V.F. Kirichenko, K. Sekigawa and N. Ejiri have studied diverse aspects of the geometry of the nearly Kählerian six-dimensional sphere.
It is also known that almost contact metric (acm-) structures are induced on oriented hypersurfaces of almost Hermitian manifolds. Many geometers observe that namely this fact determines the profound connection between the contact and Hermitian geometries. Almost contact metric structures on hypersurfaces of almost Hermitian manifolds were studied by some remarkable geometers. The works of D.E. Blair, S. Goldberg, S. Ishihara, S. Sasaki, H. Yanamoto and K. Yano are assumed classical. In the present communication, we consider acm- structures on hypersurfaces of the nearly Kählerian six-dimensional sphere.
2. The almost contact metric structure on an odd-dimensional manifold N is defined by the system of tensor fields $\{\Phi, \xi, \eta, g\}$ on this manifold, where ξ is a vector field, η is a covector field, Φ is a tensor of the type (1, 1) and $g = \langle\cdot,\cdot\rangle$ is the Riemannian metric. Moreover, the following conditions are fulfilled:

$$\eta(\xi) = 1,\ \Phi(\xi) = 0,\ \eta\circ\Phi = 0,\ \Phi^2 = -id + \xi\otimes\eta,$$

$$\langle\Phi X, \Phi Y\rangle = \langle X, Y\rangle - \eta(X)\,\eta(Y)\,,\ X, Y \in \aleph(N),$$

where $\aleph(N)$ is the module of the smooth vector fields on N [1]. As important examples of almost contact metric structures we can consider the cosymplectic structure, the nearly cosymplectic structure, the Sasakian structure and the Kenmotsu structure.
In [2], V.F. Kirichenko and I.V. Uskorev have introduced a new class of almost contact metric structure. Namely, they have defined the almost contact metric structure with the close contact form as the structures of cosymplectic type. The Kirichenko–Uskorev structure is a generalizatiom of cosymplectic and Kenmotsu structures.
3. In this communication, we present our results that are related to acm-structures induced on oriented hypersurfaces of the nearly Kählerian six-dimensional sphere. The most important of these results are the following:

1) the Cartan structural equations of the general type almost contact metric structure on an oriented hypersurface of the nearly Kahlerian six-dimensional sphere are obtained;

2) the Cartan structural equations of some important kinds of almost contact metric structures (cosymplectic, Sasaki, Kenmotsu etc.) on an oriented hypersurface of the nearly Kahlerian six-dimensional sphere are selected;

3) a characterization in terms of the type number of some important kinds of almost contact metric structures (cosymplectic, Sasaki, Kenmotsu, Kirichenko–Uskorev etc.) on an oriented hypersurface of the nearly Kählerian six-dimensional sphere is obtained;

4) a criterion of the minimality of such hypersurfaces in the terms of their type number is established;

5) a necessary and sufficient condition for an almost contact metric structure on a hypersurface of the nearly Kählerian six-dimensional sphere to be Kirichenko–Uskorev is established.

These communication is a continuation of the author's works on six-dimensional almost Hermitian manifolds (see, for example, [3], [4] and the surveys [5], [6]).

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The quasi-Sasakian hypersurfaces axiom and six-dimensional $W_1 \oplus W_2 \oplus W_4$-submanifolds of Cayley algebra

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1. The class of $W_1 \oplus W_2 \oplus W_4$-manifolds is one of so-called "big" Gray–Hervella classes [1] of almost Hermitian manifolds. This class contains all Kählerian, nearly Kählerian, almost Kählerian, quasi Kählerian, locally conformal Kählerian and Vaisman–Gray manifolds. We remind that an almost Hermitian manifold is an even-dimensional manifold M^{2n} with a Riemannian metric $g = \langle \cdot, \cdot \rangle$ and an almost complex structure J if the following condition holds

$$\langle JX,\ JY \rangle = \langle X,\ Y \rangle\,, \quad X, Y \in \aleph(M^{2n}),$$

where $\aleph(M^{2n})$ is the module of smooth vector fields on M^{2n} [2]. The fundamental form (or the Kählerian form) of an almost Hermitian manifold is determined by the relation

$$F(X,\ Y) = \langle X,\ JY \rangle\,, \quad X, Y \in \aleph(M^{2n}).$$

The Gray–Hervella condition for an arbitrary almost Hermitian structure to belong to the $W_1 \oplus W_2 \oplus W_4$ class is the following [1]:

$$\nabla_X\ (F)\ (Y, Z)\ +\ \nabla_{JX}\ \ (F)\ (JY, Z)\ =$$

$$= -\frac{1}{n-1}\left\{ \langle X, Y \rangle\ \delta\, F\,(Z) - \langle X, Z \rangle\ \delta\, F\,(Y) - \langle X, JY \rangle\ \delta\, F\ (JZ) \right\},$$

$$X, Y \in \aleph(M^{2n}).$$

In the present note, we consider six-dimensional Hermitian submanifolds of Cayley algebra. It is known that so-called Gray-Brown three-fold vector cross products in octave algebra induce almost Hermitian structures on such six-dimensional submanifolds [3], [4].
2. As it is also known, that almost contact metric structures are induced on oriented hypersurfaces of an almost Hermitian manifold [5]. We remind that the almost contact metric structure on an odd-dimensional manifold N is defined by the system of tensor fields $\{\Phi,\ \xi,\ \eta,\ g\}$ on this manifold, where ξ is a vector field, η is a covector field, Φ is a tensor of the type (1, 1) and $g = \langle \cdot, \cdot \rangle$ is the Riemannian metric [2], [5]. Moreover, the following conditions are fulfilled:

$$\eta(\xi) = 1,\ \Phi(\xi) = 0,\ \eta \circ \Phi = 0,\ \Phi^2 = -id + \xi \otimes \eta,$$

$$\langle \Phi X, \Phi Y \rangle = \langle X, Y \rangle - \eta\,(X)\,\eta\,(Y)\,,\ X, Y \in \aleph(N),$$

where $\aleph(N)$ is the module of the smooth vector fields on N. We remind that an almost Hermitian manifold M^{2n} satisfies the quasi-Sasakian hypersurfaces axiom if such a hypersurface passes through every point of this manifold. This terminology was introduced by V.F. Kirichenko in [6].
3. In [7] it has been proved that if a quasi-Kählerian manifold satisfies the quasi-Sasakian hypersurfaces axiom, then it is an almost Kählerian manifold. We generalize this statement for six-dimensional almost Hermitian submanifolds of Cayley algebra and present the main result of our communication.

Theorem. *If a six-dimensional $W_1 \oplus W_2 \oplus W_4$-submanifold of Cayley algebra satisfies the quasi-Sasakian hypersurfaces axiom, then it is an almost Kählerian manifold.*

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Moduli spaces of flat connections via knot invariants

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Let G be an algebraic, reductive, Lie group, and Σ a compact oriented surface, possibly with boundary. By classical works of Atiyah, Bott, and Goldman, the moduli space $\mathcal{M}(\Sigma, G)$ of flat connections on a principal G-bundle over Σ, or equivalently the G-character variety $Hom(\pi_1(\Sigma), G)/G$, where G acts by conjugation, admits an interesting Poisson structure. Understanding the Poisson geometry of these spaces is an open area of research, connected to several problems in mathematics and theoretical physics.
In 1996, Andersen, Mattes, and Reshetikhin proposed a topological method to use finite-type invariants of knots/links and of 3-dimensional manifolds to study $\mathcal{M}(\Sigma, G)$. They constructed Poisson algebras $\mathcal{C}(\Sigma)$ and $\mathcal{C}(\Sigma, G)$ of *colored* chord diagrams, and for every symmetric Ad-invariant tensor $t \in \mathfrak{g} \otimes \mathfrak{g}$, where $\mathfrak{g} = Lie\, G$, a Poisson homomorphism $\psi_t : \mathcal{C}(\Sigma, G) \to \mathcal{O}(\mathcal{M}(\Sigma, G))$ to the algebra of regular functions on $\mathcal{M}(\Sigma, G)$, which they proved is surjective for many important examples. Since (the completion of) the space of (*non-colored*) chord diagrams is the value space of Kontsevich's universal finite-type invariant of links, in 1998, Andersen, Mattes, and Reshtikhin have attempted to define a deformation quantization of $\mathcal{O}(\mathcal{M}(\Sigma, G))$, but their construction is non-canonical (depends on the choice of a "partition" of Σ), and therefore further progress was temporarily halted.
In 2017, Habiro, and Massuyeau have extended the Kontsevich-LMO functor (constructed in 2008 by Habiro, Massuyeau, and myself) to a functor from bottom tangles in handlebodies with values in $\overline{\mathcal{C}(\Sigma, G)}$, i.e. *colored* chord diagrams.
The main result in this talk is: *the Habiro-Massuyeau extension $Z^{\mathcal{B}}$ of the Kontsevich-LMO functor induces a $\star$-product (i.e. a deformation quantization) of $\mathcal{C}(\Sigma)$, independent of the AMR-partition of the surface.* This theorem completes Andersen-Mattes-Reshetikhin construction, and allows, by specifying various Lie groups G, to use skein theory to study the moduli spaces $\mathcal{M}(\Sigma, G)$, and their associated quantum moduli spaces.

On topological AG-quasigroup with multiple identities

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A non-empty set G is said to be a *groupoid* relatively to a binary operation denoted by $\{\cdot\}$, if for every ordered pair (a, b) of elements of G there is a unique element $ab \in G$.
If the groupoid G is a topological space and the multiplication operation $(a, b) \to a \cdot b$ is continuous, then G is called a *topological groupoid.* A groupoid G is called *medial* if it satisfies the law $xy \cdot zt = xz \cdot yt$ for all $x, y, z, t \in G$. A groupoid G is called *paramedial* if it satisfies the law $xy \cdot zt = ty \cdot zx$ for all $x, y, z, t \in G$.
If a paramedial guasigroup G contains an element e such that $e \cdot x = x(x \cdot e = x)$ for all x in G, then e is called a *left (right) identity* element of G and G is called a *left (right) paramedial loop.*
A groupoid $(G, \cdot)$ is called a *groupoid Abel-Grassmann* or *AG-groupoid* if it satisfies the left invertive law $(a \cdot b) \cdot c = (c \cdot b) \cdot a$ for all $a, b, c \in G$.
An *AG-groupoid* satisfying the identity $(a \cdot b) \cdot c = b \cdot (a \cdot c)$ or $(a \cdot b) \cdot c = b \cdot (c \cdot a)$ for all $a, b, c \in G$ is called AG^**-groupoid.* While if an *AG-groupoid* satisfying the identity $a \cdot (b \cdot c) = b \cdot (a \cdot c)$ for all $a, b, c \in G$ is called AG^{**}*-groupoid.* The notion of the (n, m)-identities and (n, m)-homogeneous isotope was introduced in [2].
The results established in this paper are related to the results of M. Choban and L. Kiriyak in [2] and to the research papers [2-4].

Theorem 1. *If $(G, +)$ is a paramedial topological groupoid and $e \in G$ is an $(1, p)$-zero, then every $(n, 1)$-homogeneous isotope $(G, \cdot)$ of the topological groupoid $(G, +)$ is a paramedial topological groupoids with $(1, np)$-identity e. Moreover, $(G, \cdot)$ is a medial, AG, AG^* and AG^{**} groupoids with $(1, np)$-identity e and $a + bc = b \cdot (a + c)$, for all $a, b, c \in G$ and $n, p \in N$.*

Proposition 1. *Let $(G, \cdot)$ be a topological AG^{**}-quasigroup $(G, \cdot)$ with an $(1, 2)$-identity e. Then the mapping $f : G \to G$, where $f(x) = xe$, is an involutive mapping, $f = f^{-1}$.*

Theorem 2. *Let $(G, \cdot)$ be a topological AG^{**}-quasigroup $(G, \cdot)$ with an $(1, 2)$-identity e and $x^2 = e$. If P is an open compact subset such that $e \in P$, then P contains an open compact AG^{**}-subquasigroup $(Q, \cdot)$ with an $(1, 2)$-identity of $(G, \cdot)$.*

Theorem 3. *Let $(G, \cdot)$ be an AG-topological quasigroup with a left identity e and $x^2 = e$ for every $x \in G$. If P is an open compact subset from G such that $e \in P$, then P contains an open compact AG-subquasigroup $(Q, \cdot)$ with a left identity e.*

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On IG-quasigroups

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In this paper we investigate quasigroups isotopic to groups (quasigroups). Among the isotopes of groups, there are distributive quasigroups, Steiner quasigroups, medial quasigroups [1], quasigroups [2], etc. The need for a detailed study of quasigroups isotopic to groups follows from the fact that, the center of a quasigroup is a quasigroup [3], and therefore the structure of quasigroups is based on quasigroups. The class of quasigroups was studied by the czech mathematicians T. Kerka, P. Nemec. It turns out that their research method is suitable and can be followed when studying quasigroups in general isotopic to groups. A groupoid $(Q, \circ)$ is called isotopic to a groupoid $(Q, \cdot)$, if there exist three permutations $\alpha, \beta, \gamma \in S(Q)$ such that $x \circ y = \gamma^{-1}(\alpha x \cdot \beta y)$ for any $x, y \in Q$. For $\gamma = \epsilon$ we obtain the principal isotope $(Q, \circ) : x \circ y = \alpha x \cdot \beta y$. It is known that any isotop is isomorphic to a principal isotope. If $(Q, +)$ is a group with zero 0, $\alpha, \beta \in S(Q)$, and $(Q, \circ)$ an isotope of the form $x \circ y = \alpha x + \beta y$, then there exists unique $a \in Q$,$\alpha_1, \beta_1 \in S(Q)$, such that $\alpha_1 0 = 0 = \beta_1 0$, $a = \alpha 0 \circ \beta 0$ and $x \circ y = \alpha_1 x + a + \beta_1 y, \quad x, y \in Q$ [4].

Definition. *The quadruple $((Q, +, 0), \alpha_1, \beta_1, a)$ is called the $IG-$form of the principal isotope $(Q, \circ) : x \circ y = \alpha x + \beta y, \quad \alpha, \beta \in S(Q), \quad x, y \in Q$, if $x \circ y = \alpha_1 x + a + \beta_1 y, \quad \alpha_1 0 = 0 = \beta_1 0$, $a = \alpha 0 \circ \beta 0$.*

Any isotope of a group is isomorphic to some of its $IG-$form. Therefore, when considering the class of quasigroups isotopic to groups, up to isomorphism, it suffices to restrict oneself to the $IG-$forms.

Theorem 1. *Let $(Q, +, 0)$ be a group with zero 0 and $(Q, \circ, e)$ a loop with identity element e. Let $\varphi, \psi \in S_0(Q)$, $\alpha, \beta \in S(Q)$, $a \in Q$, such that $\alpha x \circ \beta y = \varphi x + a + \psi y$ for all $x, y \in Q$. Then $(Q, \circ, e)$ is a group and there exist permutations $\varphi_1, \psi_1 \in S_e(Q)$ such that $\alpha x = \varphi_1 x \circ \alpha e$, $\beta y = \beta e \circ \psi_1 y$ for any $x, y \in Q$. Each of the permutations φ_1, ψ_1 can be chosen as an automorphism of the group $(Q, \circ, e)$, if only φ, ψ are the automorphisms of the group $(Q, +, 0)$, respectively.*

Theorem 2. *T-subquasigroups, homomorphic images and cartesian product of $IG-$quasigroups are $IG-$quasigroups*

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About a linearity of coordinatization ternar of an arbitrary finite projective plane

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Earlier the author proved (see [1]) the 1-1 correspondence between the finite projective plane π, some coordinatization ternary operation (x,t,y) (DK-ternar [1]) and some sharply 2-transitive loop L of permutations (which are cell permutations [1] of above mentioned DK-ternar).
In a particular case, when these loop L is a group, it can be proved the following theorem (see also [2], theorem 20.7.1).

Theorem 1. *Let G be a sharply 2-transitive group of permutations (cell permutations) on a finite set E of symbols. Then group G is isomorphic to the group of linear transformations*

$$G_K = \{\alpha \,|\, \alpha(x) = a \cdot x + b, \quad a,b \in E, \quad a \neq 0\}$$

of some near-field $K = \langle E, +, \cdot, 0, 1\rangle$, i.e.

1. $(x,t,y) = a \cdot x + b$;
2. the system $\langle E, +, 0\rangle$ is an abelian group;
3. the system $\langle E\backslash\{0\}, \cdot, 1\rangle$ is a group.

In a general case, when the loop L is not a group, the author proves the following theorem.

Theorem 2. *Let L be a sharply 2-transitive loop of permutations (cell permutations) on a finite set E of symbols. Then loop L is isomorphic to the loop of "linear transformations"*

$$G_K = \{\alpha \,|\, \alpha(x) = a \cdot x + b, \quad a,b \in E, \quad a \neq 0\}$$

of some binary loop system $K = \langle E, +, \cdot, 0, 1\rangle$, i.e.

1. $(x,t,y) = a \cdot x + b$;
2. the system $\langle E, +, 0\rangle$ is a loop;
3. the system $\langle E\backslash\{0\}, \cdot, 1\rangle$ is a loop.

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Finite cartesian product of chain connected sets in topological spaces

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In [1] the definition of connectedness of a topological space is reformulated by using the notion of chain and proved its equivalence to the standard definition of connectedness. The topological space X is connected if for every $x, y \in X$ and for any open covering $\mathcal{U}$ of X, there exists a chain in $\mathcal{U}$ that connects x and y, i.e., there exists a finite sequence $U_1, U_2, ..., U_n$ of elements of $\mathcal{U}$ such that $x \in U_1$, $y \in U_n$ and every two consecutive elements from the chain have nonempty intersection.
In [2] and [3] the definition of connectedness by using the notion of chain is generalized to the notion of chain connected set in a topological space. A set C, subset of a topological space X, is chain connected in X, if for every $x, y \in C$ and for any open covering $\mathcal{U}$ of X in X, there exists a chain in $\mathcal{U}$ that connects x and y. The topological space X is connected if and only if X is chain connected in X. If a set is connected it must be chain connected in each of its superspaces, however, the converse statement in general may not hold ([2], [3]).
Here it is proved that if A_i are chain connected sets in X_i, $\forall i \in \{1, 2, \ldots, n\}$, then the finite product $\Pi_{i=1}^{n} A_i$ is chain connected in $\Pi_{i=1}^{n} X_i$. Thus, by using the notion of chain, it is proved that the finite Cartesian product of connected spaces is a connected space. Since every connected set is chain connected in every its superspace, it is obvious that if A is a chain connected set in X and B is a connected set, then $A \times B$ is a chain connected set in $X \times B$. However, by a counterexample it is shown that $A \times B$ may be disconnected.

Keyords: connectedness, coverings, chain, chain connecteness, Cartesian product

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On some transformation groups of loops

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Let Q be a loop. The mappings $x \mapsto ax$, $x \mapsto xa$ and $x \mapsto a/x$ are called, respectively, the left, right and middle translation with a. We consider the group $TMlt(Q))$, generated by all left, right and middle translations of the loop Q, which is called the total multiplication group of Q, and the stabilizer of the unit in $TMlt(Q))$, denoted by $TInn(Q)$. The multiplication groups play an important role in the theory of normal subloops and of nilpotency of the loops [1-5]. It is known that isotopic loops have isomorphic multiplication groups [2], and that the total multiplication groups of isostrophic loops are isomorphic [4,6]. Characterizations of total multiplication groups are obtained for some classes of loops [4]. Connections between Bruck loops and middle Bol loops are established in [4]. In the middle Bol loops the multiplication group and the inner mapping group are normal subgroups of $TMlt(Q)$ and $TInn(Q)$, respectively. Characterizations of the corresponding quotient groups and of the centre of $TMlt(Q)$ are given.

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7. Computer Science

Optimal complexity algorithm for a class of problems of non-linear optimization

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The paper analyses a class of nonlinear optimization problems with special restrictions, we propose a concept for solving the auxiliary problem, for which we calculate complexity, and we also assess the maximum number of elementary operations and describe the optimal algorithm for performing numerical calculations. The study constructs an algorithm for solving nonlinear optimization problems with complexity $O\left(n^{3.5}\right)$.
Keywords: complexity of algorithms, optimization methods

Application of an asymmetric encryption algorithm based on a modified Vigenere cipher in blockchain technology

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The development of encryption, distributed database systems and information technologies generally led us to the emergence of blockchain technology, which has not only many potential, but also already implemented useful applications.
Blockchain is essentially a type of distributed data storage. The information in it is divided into a sequence of blocks connected in a "chain" using "links" - cryptographic methods. The cryptographic techniques used in the blockchain include cryptographic hash functions and asymmetric encryption algorithms. There are several basic asymmetric encryption algorithms (RSA, DES, El Gamal, DSA, ECDSA, etc.), based on different mathematical algorithms, and therefore having different characteristics. In the Bitcoin blockchain, as in the vast majority of other blockchains, the most effective for specific blockchain tasks, is the ECDSA encryption algorithm (previously the RSA algorithm was used). This algorithm is newer than, for example, the widespread RSA (which is a very strong but slow encryption algorithm). For the same security levels, ECDSA is much more compact than RSA. Accordingly, it makes it possible to process transactions faster, which is very important for the operations of the blockchain.
In the work of the authors, a fundamentally new (for blockchain systems), exotic (according to the using of non-associative algebraic systems), sufficiently strong and at the same time more compact asymmetric encryption algorithm is proposed. It is based on a modification of the Vigenere cipher using non-associative algebraic one-way functions. It allows, without losing the strength of the cipher, to reduce the length of the encryption key, as well as to reduce the time of encryption-decryption processes, which also makes it possible to process transactions faster.

Hybrid artificial neural network: combining artificial intelligence and fuzzy logic

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Artificial Neural Networks (ANNs) are used for solving the problems that have large number of observed cases. Instead of acquiring instances as prior information, fuzzy systems demand language rules. In addition, language descriptions of the input and output variables are required. There are several similarities between artificial neural network and fuzzy systems. They can be utilized to solve an issue if there is no mathematical model for the problem at hand. They have some drawbacks, which are virtually eliminated when both concepts are combined. In this research paper we will be discussing on the proposal of creating Hybrid Artificial Neural Network (HANN) by combining ANN and Fuzzy Logic (FL). In this research paper we will discuss on the problem that we are going to handle and on the solution that we are going to propose to handle the problem.

8. Education

Modele pentru activitţii de laborator la matematică în concept STEAM

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În prezentul articol sunt prezentate unele modele pentru activităţi de laborator la matematică în concept STEAM. Modelele prezentate conţin aplicaţii aferente determinării vitezei maxime de zbor şi a înălţimii unui vehicul aerian fără pilot.

Cuvinte cheie: vehicul aerian fără pilot, viteză, distanţă, timp, înălţime, unghi.

„Clasa Inversată” pentru învăţarea Programării

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Programarea, ca domeniu de studiu, este considerat deosebit de interesant și aplicativ, dar și foarte complex, dificil. Drept dovadă sunt statisticile care arată că numărul elevilor-absolvenți care aleg să susțină examenul la Informatică în sesiunea de Bacalaureat este mult mai mic decât la alte discipline. Una dintre cauze este frica elevilor față de subiectele legate de algoritmi, structuri de date și tehnici de programare, deci de Programare.
Sigur, competențele avansate de programare au la bază o pregătire matematică de calitate, dar, bunele practici arată că abordările pedagogice constructiviste de studiere a programării, care implică activ cursantul în procesul de autoinstruire, reprezintă soluții eficiente pentru obținerea de către acesta a performanțelor în domeniul Programării, chiar dacă el are unele „reticențe” față de științele exacte.
Așadar, metodele active de instruire (Instruirea în bază de Problemă, Instruirea în bază de Proiect, Instruirea prin Cercetare, Instruirea Auto-Ghidată etc.), fiind abordări constructiviste, centrate pe cel instruit, trebuie „puse în capul mesei” de instruire atunci când urmează să învățăm programarea.
Autorii au constat un succes vizibil privind utilizarea modelului „Clasa Inversată” la studierea Programării. Eficiența acestui model de instruire se explică prin faptul că el implică în cea mai mare măsură efortul cursantului și deoarece pentru domeniul Programării există o mulțime de resurse de învățare care pot fi accesate la distanță.
Cercetările realizate de autori arată că modelul „Clasa Inversată”, aplicat la studierea limbajelor de programare, este pe placul cursanților (elevi, studenți), iar rata de eșec este considerabil mai mică decât modelului tradițional de învățare. Modelul „Clasa Inversată” , transpus în cadrul studierii Programării, cuprinde următoarele **etape** şi **activităţi** în această ordine:

I. Activităţi de învăţare ale elevilor pentru acasă

- *Actualizarea cunoștințelor* (se poate realiza prin activități tip Brainstorming) presupune revizuirea cunoștințelor anterioare cu privire la tema studiată.

- *Înțelegerea materialului de învățare.* Elevii accesează sursele de învățare oferite sau sugerate de profesor. Ei studiază aceste resurse, iau notițe, căuta și colectează informații suplimentare la tema abordată.
- *Căutarea răspunsurilor la întrebări și rezolvarea problemelor simple.* Elevii încearcă să răspundă la întrebări și să rezolve probleme simple, care însoțesc resursele educaționale oferite de profesor. Acestea sunt menite să contribuie la o înțelegere mai profundă a celor studiate.
- *Reflecția asupra învățării.* După parcurgerea temei, elevii trimit individual răspunsurile la întrebările și problemele menționate anterior.

I. Activități de învățare ale elevilor (în clasă sau în regim sincron)

- *Discuții de grup.* Se lucrează în grupuri a câte 4-6 persoane. În fiecare săptămână membrii grupului sunt schimbați, astfel încât fiecare elev să poată lucra cu orice coleg din clasă, în diferite spații de învățare. Elevii discută răspunsurile la întrebările și problemele examinate acasă, prezintă informații suplimentare (găsite de fiecare) în grupul din care fac parte. Profesorul monitorizează discuția elevilor și intervine cu corectări dacă este cazul.
- *Formularea întrebărilor și oferirea răspunsurilor.* Fiecare grup pune profesorului întrebări cu referire la conținuturile neclare. La fel, elevii pot adresa întrebări profesorului în mod individual.
- *Aplicarea.* Elevii lucrează individual. Ei rezolvă probleme de consolidare, aprofundare și aplicare a celor studiate.
- *Discuție liberă.* Profesorul adresează întrebări de reflecție asupra celor studiate.

III. Activități de evaluare ale elevilor (în clasă sau în regim sincron)

Se pot realiza:

- *Teste de laborator*: profesorul cere elevilor să rezolve problemele din test prin experiment de laborator. Ulterior oferă feedback elevului și înregistrează progresul fiecăruia;
- *Evaluări autoevaluări*: profesorul determină progresul elevilor și ale grupurilor de elevi.

În cazul instruirii la distanță, pentru activitățile sincrone cu elevii cel mai indicată este utilizarea instrumentului Google Meet (cu cont de G Suite), care permite organizarea activităților educaționale online în timp real, cu toată clasa de elevi sau în grupuri. Profesorul poate iniția și menține conversații, monitoriza prezența elevilor și urmări activitatea elevilor prin conexiunile camerelor video. În calitate de mediu digital de învățare se recomandă folosirea unui Sistem de management al instruirii și al conținuturilor de învățare, gen Moodle, dar poate fi valorificat și Google Classroom, Microsoft 365 etc. Prin intermediul platformei de instruire elevii transmit rezolvările exercițiilor/problemelor (examinate în clasă sau acasă), accesează anunțurile, sarcinile propuse de profesor, întrebările puse în discuție, iar profesorul urmărește progresul lor la fiecare unitate de învățare.

Constatări și concluzii

De rând cu formarea și dezvoltarea abilităților de algoritmizare, programare, modelul „Clasa Inversată", pliată pe studierea Programării, provoacă curiozitatea, dezvoltă creativitatea, gândirea și raționamentul la nivel înalt, comunicarea, colaborarea, spiritul de inițiativă. Ea responsabilizează elevii, le conferă importanță, îi implică în activități de învățare (cu posibilitatea de a studia independent). Abordată cu tehnologii informaționale incluse în mediul de învățare, „Clasa Inversată" face ca elevii să nu se simtă înstrăinați, stresați. Ea le oferă timp să interacționeze, să colaboreze cu colegii și cu profesorul pentru a exersa mai mult, deci le creează o realitate plină de autonomie, responsabilitate și șansa ca ei să devină actori reali și activi ai procesului de învățare.

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Determinarea valorilor exacte ale unor integrale improprii de la funcţii ce nu au primitivă

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a) Aplicând metoda substituției și proprietățile integralei definite se determină valorile următoarelor integrale:

$$\int_0^{\infty} \frac{\ln x}{x^2+1}\mathrm{d}x = 0, \int_0^{\infty} \frac{\ln x}{x^2+a^2}\mathrm{d}x = \frac{\pi \ln a}{2a}, a > 0, \int_0^{\frac{\pi}{2}} \ln(\mathrm{tg}x)\mathrm{d}x = 0.$$

b) Aplicând metoda substituției, prin reducere la o ecuație liniară ce conține ca necunoscută integrala dată inițial, se obțin următoarele valori:

$$\int_0^{\frac{\pi}{2}} \ln(\sin x)\mathrm{d}x = \int_0^{\frac{\pi}{2}} \ln(\cos x)\mathrm{d}x = -\frac{\pi \ln 2}{2}, \int_0^{\pi} \ln(\sin x)\mathrm{d}x = -\pi \ln 2.$$

c) Aplicând metoda substituției și integrarea prin părți se deduce, că

$$\int_0^1 \frac{\arcsin x}{x}\mathrm{d}x = \frac{\pi \ln 2}{2}$$

d) Aplicând metoda substituției și dezvoltarea funcției în serie de puteri se determină, că

$$\int_0^{\infty} \frac{x}{1-\mathrm{e}^x}\mathrm{d}x = \int_0^1 \frac{\ln(1-x)}{x}\mathrm{d}x = -\frac{\pi^2}{6}, \int_0^1 \frac{\ln(1-x^a)}{x}\mathrm{d}x = -\frac{\pi^2}{6a}, a \neq 0.$$

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Construcţia triunghiului fiind date trei puncte speciale

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În acest articol este propus un set de probleme geometrice de construcție în care se cere de restabilit triunghiul, fiind dată o ternă de puncte necoliniare, alcătuită din puncte de aceeași natură.

Cuvinte cheie: Triunghi, cerc, cerc înscris, cerc circumscris, problemă de construcție .

Este greu de apreciat rolul problemelor de construcție în dezvoltarea matematică a elevilor. Atât după conținut, cât și după metodele de rezolvare, astfel de probleme nu numai că stimulează acumularea unor performanțe geometrice concrete, dar dezvoltă și capacitatea de a-și imagina o figură sau alta și mai mult de cât atât, de a putea opera în gând cu elementele acestei figuri. Problemele geometrice de construcție ajută elevilor să înțeleagă apariția diferitor figuri geometrice, posibilitatea de a le transforma, iar toate acestea constituie importante premise pentru dezvoltarea gândirii spațiale la elevi și studenți.Astfel de probleme în mare măsură dezvoltă gândirea logică și intuiția geometrică.

Prin puncte speciale pentru triunghiul arbitrar ABC vom înțelege o ternă de puncte necoliniare $[E_1, E_2, E_3]$, care în mod univoc determină triunghiul ABC . Așa de exemplu, sunt mijlocurile laturilor, picioarele înălțimilor, simetricele ortocentrului în raport cu laturile triunghiului, etc. Astfel de puncte se mai numesc puncte de aceeași natură.

Unul din scopurile studierii geometriei constă în formarea deprinderilor de a rezolva problemele geometrice. Aceste deprinderi pot fi formate doar în rezultatul rezolvării a unui număr cât mai mare de astfel de probleme. Pentru a educa la elevi(studenți) dorința de arezolva probleme geometrice apar un șir de cerințe referitor la problemele propuse pentru a fi rezolvate. Printre aceste cerințe sunt: problemele trebuie să fie laute din practică, din natură, din viața cotidiană, să fie interesante, atractive, etc.

Reeșind din cele enumerate de mai sus, propunem să rezolvăm cu elevii(studenții) un set de problem de construcție.

De restabilit triunghiul, fiind date:

a) mijlocurile laturilor;

b) picioarele înălțîmilor;

c) simitricile centrului cercului circumscris triunghiului în raport cu laturile acestui triunghi;

d) punctele simetrice centrului cercului înscris în raport cu laturile triunghiului;

e) simetricele ortocentrului în raport cu mijlocurile laturilor triunghiului;

f) punctele de tangență a cercului înscris în acest triunghi cu laturile triunghiului;

g) mijlocurile segmentelor ce unesc ortocentrul cu vîrfurile triunghiului;

h) punctele de intersecție a prelungirilor bisectoarelor triunghiului cu cercul circumscris acestui triunghi;

i) picioarele bisectoarelor triunghiului isoscel.

Propunem pentru a fi rezolvate de sine stătător următoarele probleme:

1 Punctele E_1, E_2 și E_3 sunt primele puncte de intersecție a bisectoarelor unghiurilor triunghiului ABC cu cercul înscris în acest triunghi. De construit triunghiul ABC , fiind date punctele E_1, E_2 şi E_3.

2 Prelungirile înălțimilor triunghiului ABC intersectează cercul ω circumscris acestui triunghi în punctele Q_1, Q_2 şi Q_3 corespunzător. De construit triunghiul ABC, fiind date punctele Q_1, Q_2 şi Q_3.

3 De construit triunghiul ABC, fiind date centrele O_1, O_2 şi O_3 a cercurilor exînscrise triunghiului ABC. *Indicație. Demonstrați, că picioarele înălțimilor triunghiului $O_1O_2O_3$ coincid cu vârfurile triunghiului ABC.*

4 De construit triunghiul dreptunghic ABC , fiind date picioarele bisectoarelor.

Orice problemă rezolvată de către elevi (studenți) reprezintă pentru ei o descoperire. Teoria și practica învățării prin descoperire (discovery learning),după cum se menționează în [1], reprezintă un ansamblu de procese foarte complexe, bazate pe procedeuri euristice și de cercetare care-i determină pe elevi (studenți) să descopere prin ei însăși noi adevăruri,să rezolve însăși probleme.Astfel învățarea prin descoperire denotă un mai mare grad de eficiență intelectuală, cultivă o motivație interioară a învățării.

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Abordări metodologice în utilizarea triangulării Delaunay şi a diagramei Voronoi

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În această lucrare vom descrie algoritmi fundamentali pentru găsirea locației fizice a unui nod într-o rețea [1] care sunt bazați pe triangularea Delaunay și a diagramei Voronoi.
Putem rezuma problema triangulării unei mulțimi de puncte S în plan prin găsirea unei organizări $DT(S)$ astfel încât să nu existe nici un punct S în cercurile circumscrise triunghiurilor din $DT(S)$. Algoritmul Delaunay produce triunghiuri aproape echiunghiulare și are o complexitate $O(n\log(n))$. Pentru S, o mulțime formată dintr-un număr n din plan, orice triangulare a lui S este formată din $2n-2-k$ triunghiuri și $3n-3-k$ muchii, unde n este numărul de puncte din mulțimea S iar k este numărul de puncte care fac parte din învelitoarea convexă.
Triangularea Delaunay a unui ansamblu de puncte date S este definită de următoarea proprietate: *orice cerc circumscris unuia dintre triunghiurile componente ale triangulării nu conține nici un alt punct al ansamblului înăuntrul său.*
O triangulare Delaunay este în realitate structura duală în sens de graf asociate unei diagrame Voronoi. Triangulația Delaunay și Diagrama Voronoi – două noțiuni duale. Diagrama Voronoi pentru o mulțime de puncte reprezintă descompunerea planului în celule care conțin punctele cele mai apropiate de punctele date inițial, numite locații. Pentru fiecare latură a unei celule Voronoi,

există o latură în triangularea Delaunay. Astfel, triangularea Delaunay reprezintă un graf dual al diagramei Voronoi pentru aceeași mulțime de puncte, care are drept vârfuri locațiile și câte o față corespunzătoare fiecărei celule Voronoi [2].
În această lucrare a fost elaborată o aplicație care ne permite implementarea algoritmului de localizare optimă a senzorilor într-o rețea de senzori wi-fi la baza căreia stă triangulația Delaunay și diagrama Voronoi.

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Impactul metodelor interactive în predarea unităţii de curs *Programarea aplicaţiilor client-server*

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Metodele active și cele interactive permit partajarea informațiilor, realizarea feedbackului, rezolvarea colectivă a problemelor apărute, simularea situațiilor educaționale, evaluarea comportamentului propriu și a acțiunilor altor participanți, crearea unei atmosfere reale de cooperare pentru rezolvarea problemelor. Datorită metodelor interactive, studenții își formează competențe profesionale, își dezvoltă gândirea analitică, își mobilizează puterile cognitive, trezindu-și astfel interesul pentru noi cunoștințe și dezvăluindu-și creativitatea. Această comunicare reflectă experiența utilizării metodelor interactive cu studenții la studierea unității de curs PACS (Programarea Aplicațiilor Client-Service).

Cuvinte-cheie: metode interactive, discuții de grup, situații educaționale, proces didactic, sarcini creative.

Elaboration of the technology of creation on interactive manuals using the Google Suite for Education package and presentation capabilities of the LATEX programm language

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In the field of education (both primary and secondary and higher), there is the problem of the relevance of conventional (printed) manuals, their corresponding to the current moment of time, the level of development of computer technology and ways of communication. Simply, modern printed textbooks in some areas (such as computer science) are rapidly becoming outdated. On the other hand, it is not reasonable to print new manuals every year, especially if only a small amount of content needs to be changed in them. So we come to the idea of electronic interactive manuals that can be placed on various media, or can be accessed on special educational portals.

The main thing in these textbooks is their interactivity, that is, the ability to easily and quickly replace part of the content in accordance with the requirements of the educational process. In addition, the multimedia component of such a manual is important (which is absent in an ordinary paper textbook).
The most basic question is how to create such manuals? The proposed work uses the applications Google Classroom, Google Foto, Jamboard, etc. from the Google Suite for Education package and presentation technologies based on the LATEX programming language. These applications and technologies make it possible to form the content of the textbook quite flexibly, with the possibility of simple and quick replacement of the content (up to individual lessons or their parts), and with full visualization of certain topics (which is especially important, for example, in physics).
The result of the work is a technology for creating interactive manuals on any selected topic (for example, in mathematics or physics) with a modular thematic structure. This technology allows the teacher to use such a manual when teaching both traditionally or for distance learning. In addition, this manual can be easily modified according to curriculum updates and specific teacher goals.

Unele aspecte privind evoluţia conceptelor de combinatorică aplicate în educaţia matematică

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Conceptele de combinatorică sunt aplicabile în educația matematică pentru toate vârstele. De-a lungul istoriei au fost elaborate tratate privind diverse capitole ale combinatoricii, iar începând cu sfârșitul secolului al XX-lea au fost dezvoltate metode teoretice puternice. În context educațional elementele de combinatorică permit realizarea de relații intradisciplinare, conectând algebra, geometria, teoria probabilităților, statistica, dar și relații interdisciplinare cu fizica, informatica, științele naturii ș.a. Istoria evoluției noțiunilor de combinatorică abundă de numeroase reprezentări, atât numerice cât și geometrice. Una dintre cele mai vechi părți ale combinatoricii este considerată teoria grafurilor, iar aplicațiile moderne vizează deja teoria fractalilor. În opinia noastră, este importantă familiarizarea elevilor și studenților cu unele tratate de combinatorică care țin de palindroame, numerele perfecte, triunghiul lui Pascal, binomul lui Newton, numerele Stirling, numerele Bell, triunghiul lui Sierpinski ș.a. pentru a dezvălui evoluția conceptelor și aplicațiile lor în diverse domenii. Investigația realizată a avut drept obiectiv analiza logico-didactică a conținuturilor în vederea stabilirii zonelor de interferenţă a diverse discipline, posibilităților de transgresare între trepte, nivele, cicluri de învățământ și a facilitării transpunerii didactice în cheia abordării STEAM a învățământului matematic.

Aspecte privind procesul educaţional online la disciplina "Grafică asistată de calculator"

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O provocare majoră pentru toţi actorii sistemului educaţional este constituită de dinamica schimbărilor ce au loc în lume dar şi în Republica Moldova. Promovarea şi acceptarea paradigmei

postmoderne, bazate pe umanism şi constructivism, abordarea educaţiei din perspectiva celui ce învaţă şi desfăşurarea procesului de învăţământ din perspectiva pedagogiei axate pe competenţe sunt doar câteva dintre noile imperative. Mai nou, pandemia cauzată de Covid-19 a dat peste cap multe activităţi tradiţionale din societatea noastră, impunând schimbări cardinale în diverse domenii, inclusiv în Educaţie. Activitatea instituţiilor de învăţământ a fost suspendată pentru un termen relativ lung. Autorităţile şi colectivele pedagogice căutând ieşire din această situaţie, au încercat să utilizeze posibilităţile oferite de programele on-line, un proces absolut nou pentru Republica Moldova. Prezenta lucrare reflectă aspectele legate de predarea-învățarea-evaluarea online a disciplinelor din domeniul informaticii, pe exemplul disciplinei “Grafica asistată de calculator”. Sunt expuse punctele tari și slabe ale procesului educațional online, precum și rezolvarea dilemei privind software-ul necesar pentru lecțiile de informatică.

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