DOI 10.1007/s10559-018-0099-8
Cybernetics and Systems Analysis, Vol. 54, No. 6, November, 2018

EXISTENCE OF LYAPUNOV–KRASOVSKII FUNCTIONALS FOR STOCHASTIC FUNCTIONAL DIFFERENTIAL ITO–SKOROKHOD EQUATIONS UNDER THE CONDITION OF SOLUTIONS' STABILITY ON PROBABILITY WITH FINITE AFTEREFFECT

I. V. Yurchenko[1†] **and V. K. Yasynskyy**[1‡]

UDC 519.217:519.718:519.837

Abstract. *In the paper, it is established that Lyapunov–Krasovskii functionals with definite properties exist for dynamic systems of random structure with finite prehistory and with the property of one or another probability stability.*

Keywords: *systems of random structure, aftereffect, stability, Lyapunov–Krasovskii functionals.*

INTRODUCTION

The main studies in stability and optimal stabilization for deterministic systems of ordinary differential equations and differential equations with aftereffect are those by Krasovskii, Letov and Lidskii [1, 2], Kolmanovskii, Nosov, and Shaikhet [3, 4], as well references therein.

Samoilenko and Perestyuk [5] described systematically the possibility of taking into account impulse perturbations in differential equations. This situation is also analyzed in detail by Tsar'kov and Sverdan in [6] not only for differential equations but also for difference equations.

The influence of Markov perturbations on stability of dynamic systems is described by Korolyuk, Limnios [7], Andreeva, Kolmanovskii, Shaikhet [3], Khas'minskii [8], Katz, Krasovskii [9], Gorelik [10], Kolmanovskii, Khas'minskii [11], Tsar'kov, Yasynskyy [12, 13], by other authors [14–17], by Skorokhod in his fundamental monograph [18], as well as in the references in these studies. The Lyapunov functions method in problems of stability and stabilization of systems of random structure is described by Katz in the monograph [19]. Stability of an autonomous dynamic system with fast Markov switching was studied by Korolyuk in [20–22].

In the paper, we will consider and solve the problem about behavior of a dynamic system in the presence of Markov perturbations (parameters), which possesses the property of global asymptotic stability in probability and, in case of linear systems, the property of exponential stability in quadratic mean.

The idea of the asymptotics of the solution of the above-mentioned problem is based on the method of Lyapunov's functions and functionals [11]. For dynamic systems with aftereffect, this idea was implemented in [3, 6, 12–14, 19, 23–28].

The present paper develops the ideas and methods of the analysis of global asymptotic stability in the interpretation of the stochastics of impulse dynamic systems that consider Markov perturbations.

---

[1]Yuriy Fedkovych Chernivtsi National University, Chernivtsi, Ukraine, [†]*i.yurchenko@chnu.edu.ua*; [‡]*v.yasynskyy@chnu.edu.ua*. Translated from Kibernetika i Sistemnyi Analiz, No. 6, November–December, 2018, pp. 119–133. Original article submitted January 20, 2018.

1. PROBLEM STATEMENT

Let on a probability basis $(\Omega, F, P, \mathcal{F}\}, \{\mathcal{F} \equiv F_t \subset F, t \geq 0\}$, a dynamic system of random structure (DSRS) with finite aftereffect in the form of a stochastic functional-differential Ito–Skorokhod equation be defined for $t \geq t_0 = 0$

$$dx(t) = a(t, x_t, \xi(t)) + b(t, x_t, \xi(t))\,dw(t) + \int_U c(t, x_t, \xi(t), u)\,\tilde{\nu}(du, dt) \tag{1}$$

with the initial condition

$$x(t+\theta, \omega)\big|_{t=0} = \varphi(\theta, \omega), \ \ \xi(t)\big|_{t=0} = y_0 \in \mathbf{Y}. \tag{2}$$

Here, $x(t) \equiv x(t, \omega) \in \mathbf{R}^n$, $x_t \equiv \{x(t+\theta), -\tau \leq \theta \leq 0, 0 \leq \tau \leq \infty\} \in \mathbf{D}([-\tau, 0])$, where $\mathbf{D}([-\tau, 0])$ is the Skorokhod space [5] of right-continuous functions having left-side limits [18, 20]; $a: \mathbf{R}_+ \times \mathbf{D}([-\tau, 0]) \times \mathbf{Y} \to \mathbf{R}^n$; $b: \mathbf{R}_+ \times \mathbf{D}([-\tau, 0]) \times \mathbf{Y} \to \mathbf{R}^n$; $c: \mathbf{R}_+ \times \mathbf{D}([-\tau, 0]) \times \mathbf{Y} \times \mathbf{U} \to \mathbf{R}^n$; $w(t) \equiv w(t, \omega): \mathbf{R}_+ \times \Omega \to \mathbf{R}^1$ is a standard Wiener process [17]; $\tilde{\nu}(t, A) = \nu(t, A) - t\,\Pi(A)$ is a centered Poisson measure; $\xi(t)$ is a simple Markov scheme with finite number of states $\xi(t) \equiv \xi(t, \omega) \in \mathbf{Y} \equiv \{y_1, y_2, \ldots, y_k\}$, which is defined by transition probabilities for each $t \geq s \geq 0$ [17]

$$P\{\xi(t+\Delta t) = y_j \mid y(t) = y_i\} = q_{ij}\Delta t + o(\Delta t), \tag{3}$$

$$P\{y+(s) \equiv y_i, t \leq s \leq t+\Delta t \mid y(t) = y_i\} = 1 - q_i \Delta t + o(\Delta t). \tag{4}$$

Let us consider the case where at the moment $s > 0$ of variation in the structure of system (1), (2) there is a random jump in the phase vector $x(s-0) = x$, $x(s) = z$, for which the conditional density $p_{ij}(\tau, z / x)$ is specified, i.e.,

$$P\{x(s) \in [z, z+dz) \mid x(s-0) = x\} = p_{ij}(s, z / x)dz + o(dz). \tag{5}$$

Remark 1. The stochastic functional-differential equation (1) is a formal notation of the following stochastic integral Ito–Skorokhod equation:

$$x(t) = \varphi(0) + \int_0^t a(s, x_s, \xi(s))\,ds + \int_0^t b(s, x_s, \xi(s))dw(s) + \int_0^t \int_U c(s, x_s, \xi(s), u)\,\tilde{\nu}(du, ds), \tag{6}$$

where the first integral is a Riemann integral, the second one is an Ito integral, and the third one is a Skorokhod integral introduced with respect to the Poisson measure [18].

Remark 2. In what follows, it is natural to suppose that the condition about pairwise independence of $\xi(t, \omega), w(t)$, and $\tilde{\nu}(t, A)$ is satisfied [18].

Assume that functionals a, b and c satisfy the Lipschitz property at any finite domain $\|x_t\| < H$, $\|x_t\| \equiv \sup_{-\tau \leq \theta \leq 0} |x(t+\theta)|$, for any $\varphi, \psi \in C([-\tau, 0])$

$$|a(t, \varphi, y) - a(t, \psi, y)| + |b(t, \varphi, y) - b(t, \psi, y)| + \int_U |(c(t, \varphi, y, u) - c(t, \psi, y, u))\Pi(du)| \leq L\|\varphi - \psi\|, \tag{7}$$

as well as the condition of boundedness of the coefficients

$$|a(t, \varphi, y)| + |b(t, \varphi, y)| + \int_U |c(t, \varphi, y, u)|\Pi(du) \leq L(1 + \|\varphi\|) \tag{8}$$