Approximation of systems with delay their stability

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In this paper, the schemes of approximation of linear systems with delays by special systems of ordinary differential equations are considered and the connections between their solutions are investigated. This allowed us to build algorithms for studying the stability of linear systems with delay.

Consider the initial problem for a linear system of differential-difference equations

$$\frac{dx}{dt} = Ax(t) + \sum_{i=1}^{k} B_i x(t - \tau_i), \tag{1}$$

$$x(t) = \varphi(t), t \in [-\tau, 0], \tag{2}$$

where $A, B_i, i = \overline{1,k}$ fixed $n \times n$ matrix $x \in R^n$, $0 < \tau_1 < \tau_2 < ... < \tau_k = \tau, \varphi(t) \in [-\tau, 0]$.

Let us correspond to the initial problem (1) – (2) the system of ordinary differential equations [1-3]

$$\frac{dz_0(t)}{dt} = A(t)z_0(t) + \sum_{i=1}^{k} B_i z_{l_i}(t), l_i = [\frac{\tau_i m}{\tau}],$$

$$\frac{dz_j(t)}{dt} = \mu(z_{j-1}(t) - z_j(t)), j = \overline{1,m}, \mu = \frac{\mu}{\tau}, \mu \in N,$$

with initial conditions

$$z_j(0) = \varphi(-\frac{\tau j}{m}), j = \overline{0,m}.$$

Theorem [1]. *If the zero solution of the system with delay (1) is exponentially stable (not stable), then there is $m_0 > 0$ such that for all $m > m_0$, the zero solution of the approximating system (3) is also exponentially stable (not stable).*

If for all $m > m_0$ zero solution of approximation system (3) is exponentially stable (not stable) then the zero solution of the system with a delay (1) is exponentially stable (not stable).

It follows from Theorem that for a sufficiently large m the asymptotic stability (instability) of the zero solution of a linear system with a delay is equivalent to the asymptotic stability (instability) of the zero solution of the system of approximate ordinary differential equations.

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