METRIZABILITY OF PARTIAL METRIC SPACES

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Abstract. We analyze relationship between partial metric spaces and several generalized metric spaces. We first establish that the perfectness of an arbitrary partially metric space X is equivalent to each of the following properties: developability, semi-stratifiability, X has a σ-discrete closed network, X is a β-space with T_1. Using this fact we obtain that the metrizability of a partial metric space X is equivalent to the stratifiability of X or to the fact that X is a perfect regular paracompact space. Moreover, we give examples that indicate the essentiality of certain conditions in the previous two results. Finally, we show that the statement "every perfectly normal separable partial metric space is metrizable" is independent of ZFC, similarly as for the Normal Moore Space Problem.

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