EXISTENCE AND STABILITY OF TRAVELING WAVES IN PARABOLIC SYSTEMS OF DIFFERENTIAL EQUATIONS WITH WEAK DIFFUSION

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Consider the equation [1–3]

$$\frac{\partial u}{\partial t} = i\omega_0 u + \varepsilon\left[(\gamma + i\delta)\frac{\partial^2 u}{\partial x^2} + (\alpha + i\beta)u\right] + (d_0 + ic_0)u^2\overline{u} \tag{1}$$

with periodic boundary condition

$$u(t, x + 2\pi) = u(t, x), \tag{2}$$

where ε is a small positive parameter.

Theorem 1. *If* $\omega_0 > 0$, $\alpha > 0$, $\gamma > 0$, $d_0 < 0$ *and the condition* $\alpha > \gamma n^2$ *is satisfied for some* $n \in \mathbb{Z}$. *Then for some* $\varepsilon_0 > 0$, $0 < \varepsilon < \varepsilon_0$, *the periodic on* t *solutions*

$$u_n = u_n(t, x) = \sqrt{\varepsilon} r_n \exp(i(\chi_n(\varepsilon)t + nx)) + O(\varepsilon)$$

of the boundary value problem (1), (2) exists. Here $r_n = \sqrt{(\alpha - n^2\gamma)\,|d_0|^{-1}}$, $\chi_n(\varepsilon) = \omega_0 + \varepsilon\beta + \varepsilon c_0 r_n^2 - \varepsilon\delta n^2$, $n \in \mathbb{Z}$.

These solutions are exponentially orbitally stable if and only if the condition

$$(d_0 r_n^2 - \gamma k^2)^2(\gamma^2 k^2 + \delta^2 k^2 - 2\gamma d_0 r_n^2 - 4\gamma^2 n^2 - 2\delta c_0 r_n^2) > 4\gamma^2 n^2 (c_0 r_n^2 - \delta k^2)^2$$

is satisfied for all $k \in \mathbb{Z}\backslash\{0\}$.

References

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