EXISTENCE AND STABILITY OF CYCLES IN PARABOLIC SYSTEMS WITH SMALL DIFFUSION

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The existence of countably many cycles in hyperbolic systems of differential equations with transformed argument were considered in [6]. The existence and stability of an arbitrarily large finite number of cycles for the equation of spin combustion with delay were considered in [7]. We study the existence and stability of an arbitrarily large finite number of cycles for a parabolic system with delay and weak diffusion. Similar problems for partial differential equations were studied in numerous works (see, e.g., [1 – 8]).

1. Traveling waves for parabolic equations with weak diffusion. Consider a system

$$\frac{\partial u_1}{\partial t}=\varepsilon\gamma\frac{\partial^2 u_1}{\partial x^2}-\varepsilon\delta\frac{\partial^2 u_2}{\partial x^2}-\omega_0 u_2+\varepsilon(\alpha u_1-\beta u_2)+(d_0u_1-c_0u_2)(u_1^2+u_2^2),$$

$$\frac{\partial u_2}{\partial t}=\varepsilon\gamma\frac{\partial^2 u_2}{\partial x^2}+\varepsilon\delta\frac{\partial^2 u_1}{\partial x^2}+\omega_0 u_1+\varepsilon(\alpha u_2+\beta u_1)+(d_0u_2+c_0u_1)(u_1^2+u_2^2) \qquad (1)$$

with periodic condition

$$u_1(t,x+2\pi)=u_1(t,x),\quad u_2(t,x+2\pi)=u_2(t,x), \qquad (2)$$

where ε is a small positive parameter, $\omega_0>0$, $\alpha>0$, $\gamma>0$, $d_0<0$.

Passing to the complex variables $u=u_1+iu_2$ and $\bar u=u_1-iu_2$, we arrive at the equation

$$\frac{\partial u}{\partial t}=i\omega_0u+\varepsilon\left[(\gamma+i\delta)\frac{\partial^2 u}{\partial x^2}+(\alpha+i\beta)u\right]+(d_0+ic_0)u^2\overline{u}. \qquad (3)$$

We investigate the existence and stability of the wave solutions of problem (1), (2). The solution of equation (3) is sought in the form of traveling wave $u=\theta(y)$, $y=\sigma t+x$, where the function $\theta(y)$ is periodic with period 2π. We arrive at the equation

$$\sigma\frac{d\theta}{dy}=i\omega_0\theta+\varepsilon\left[(\gamma+i\delta)\frac{d^2\theta}{dy^2}+(\alpha+i\beta)\theta\right]+(d_0+ic_0)\theta^2\overline{\theta}.$$

By the change of variables $\frac{d\theta}{dy}=\theta_1$, this equation is reduced to the following system:

$$\frac{d\theta}{dy}=\theta_1,\quad \sigma\theta_1=i\omega_0\theta+\varepsilon\left[(\gamma+i\delta)\frac{d\theta_1}{dy}+(\alpha+i\beta)\theta\right]+(d_0+ic_0)\theta^2\overline{\theta}.$$

The periodic solution of problem (1), (2) takes the form

$$u_1=\sqrt{\varepsilon}r_n\cos(\chi_n(\varepsilon)t+nx),\quad u_2=\sqrt{\varepsilon}r_n\sin(\chi_n(\varepsilon)t+nx),\quad n\in\mathbb{Z}, \qquad (4)$$

where $r_n=\sqrt{(\alpha-n^2\gamma)\,|d_0|^{-1}}$, $\chi_n(\varepsilon)=\omega_0+\varepsilon\beta+\varepsilon c_0r_n^2-\varepsilon\delta n^2$, $n\in\mathbb{Z}$.

The following statement is true:

Theorem 1. *Let $\omega_0>0$, $\alpha>0$, $\gamma>0$, $d_0<0$ and let the inequality $\alpha>\gamma n^2$ be true for some integer n. Then there exists $\varepsilon_0>0$ such that, for $0<\varepsilon<\varepsilon_0$, problem (1), (2) has solutions (4) periodic in t.*

The problems of existence and stability of traveling waves in the parabolic system with retarded argument and weak diffusion are investigated.

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